GMS 90 Exam 2 Solutions

Nov 7, 2011

See solutions to problems 1–14 in Hawkes Learning System.

15. (15 pts) Solve the equation

$$14 - 3x = 6x$$

for the real number x. Justify each step by referring to an appropriate algebraic property. Be sure not to skip or combine steps.

$14 - 3x = 6x$	
$14 + (-3x) = 6x$	definition of subtraction
$(14 + (-3x)) + 3x = 6x + 3x$	addition property of equality
$14 + (-3x + 3x) = 6x + 3x$	associativity of addition
$14 + 0 = 6x + 3x$	inverse property of addition
$14 = 6x + 3x$	identity property of addition
$14 = (6 + 3)x$	distributivity of multiplication over addition
$14 = 9x$	arithmetic
$\frac{1}{9}14 = \frac{1}{9}(9x)$	multiplication property of equality
$\frac{14}{9} = \frac{1}{9}(9x)$	arithmetic
$\frac{14}{9} = \left(\frac{1}{9}9\right)x$	associativity of multiplication
$\frac{14}{9} = 1x$	inverse property of multiplication
$\frac{14}{9} = x$	identity property of multiplication

I'll check my answer to be sure:

$$14 - 3\,\frac{14}{9} = 14 - \frac{14}{3} = \frac{42}{3} - \frac{14}{3} = \frac{28}{3}$$

$$6\,\frac{14}{9} = 2\frac{14}{3} = \frac{28}{3} \qquad \checkmark$$

16. (a) (6 pts) Define when a linear equation in one variable is a conditional equation, an identity, or a contradiction.

    A linear equation in one variable is a conditional equation if it has a unique solution, an identity if any real number is a solution, and a contradiction if it has no real solution.

    (b) (9 pts) Give an example each of the kind of linear equation in one variable. In each case, explain how you know that your equation is of the required type.

    (i) A conditional equation

    $2x = 4$ is a conditional equation because its only solution is $x = 2$. This is easy to see by dividing both sides by 2.

(ii) An identity

$3x - 3 = 3(x - 1)$ is an identity because using distributivity on the right hand side allows us to rewrite it as $3x - 3 = 3x - 3$, and it is clear that any real number satisfies this equation.

(iii) A contradiction

$x + 1 = x + 2$ is a contradiction, because no matter what real number we try to substitute for x, the right hand side will always be one more than the left hand side.

17. (10 pts) **Extra credit problem.** Recall the game of Fibonacci Nim, which goes as follows. Two players, let's call them Alice and Bob, play against each other by taking turns removing objects, say beans, from a pile. Alice goes first and she may remove as many beans as she wants, except for all of them. Bob can then remove up to twice the number of beans Alice has just removed. Now it is Alice's turn, and she can remove up to twice the number of beans Bob has just removed. And so on, each time a player can remove up to twice the number of beans the other player removed in the preceding turn. The player that removes the last bean wins. For example, if the pile initially has 5 beans, and Alice removes 1 on her first move, then Bob can take 1 or 2. If Bob takes 2, Alice can now take 1–4, except of course that there are only 2 beans left. By taking those 2, Alice wins the game.

Show that if the pile initially has 10 beans, Alice can win no matter what Bob does. (Hint: Try 2 beans, 3 beans, 4 beans, etc to understand who has a winning strategy in each case.)

Starting with 2 beans, Alice is bound to lose, because her only legal move is to take one of them, and then Bob will take the other.

Starting with 3 beans, Alice will again lose, because whether she starts by taking one or two beans, Bob can take the rest.

Starting with 4 beans, Alice has a winning strategy. If she takes one bean, than Bob will face a pile of 3, and he can only take one or two. Whether he takes one or two, Alice can take the rest. In fact, notice that the reason Bob loses this game is the same why Alice would lose if the pile initially had only 3 beans.

Starting with 5 beans, Alice will lose. If she takes two, three, or four, Bob will take the rest. If she takes one, then Bob has a pile of 4, and he can win using the same strategy Alice would use to win if starting with 4 beans.

Starting with 6 beans, Alice has a winning strategy. She can take just one bean, and put Bob in the same kind of losing situation she was in when starting with 5 beans.

Starting with 7 beans, Alice again has a winning strategy. She can take two beans, and put Bob in the same kind of losing situation she was in when starting with 5 beans.

Starting with 8 beans, Alice will lose. If she takes three or more beans, Bob can take the rest. If she takes one or two, she leaves Bob with the same kind of winning situation she was in when they started with 6 or 7 beans.

Starting with 9 beans, Alice has a winning strategy. If she takes one bean, she will put Bob in the same kind of losing situation she was in when they started with 8 beans.

Finally, starting with 10 beans, Alice again has a winning strategy. If she takes two, Bob will face a pile of 8, and will lose for the same reason Alice lost when they started with 8.

Note that you can continue this kind of analysis as long as you want. Eventually, things get a little complicated to do it by hand, but one could use a computer to investigate the game up to many beans.

Prof. Salamon at the San Diego campus of SDSU used to challenge SDSU students by posting the Problem of the Fortnight. POF #59 posted on Feb 15, 2008 is about a Fibonacci Nim game which starts with 40 beans. Check it out at `www.cs.sdsu.edu/news/fortnight/past.html`.