GMS 91 Exam 1 Solutions

Oct 2, 2012

See solutions to problems 1–11 in Hawkes Learning System.

12. (10 pts) Fully simplify the following complex fraction

$$\frac{\frac{3}{x^2+x-2}-\frac{5}{x+2}}{\frac{1}{x^2-1}+\frac{9}{yx-y}}.$$

Check your answer.

$$\begin{aligned}
\frac{\frac{3}{x^2+x-2}-\frac{5}{x+2}}{\frac{1}{x^2-1}+\frac{9}{yx-y}} &= \frac{\frac{3}{(x+2)(x-1)}-\frac{5}{x+2}}{\frac{1}{(x+1)(x-1)}+\frac{9}{y(x-1)}} \\
&= \frac{\frac{3}{(x+2)(x-1)}-\frac{5}{x+2}}{\frac{1}{(x+1)(x-1)}+\frac{9}{y(x-1)}} \cdot \frac{(x+2)(x+1)(x-1)y}{(x+2)(x+1)(x-1)y} \\
&= \frac{3(x+1)y-5(x+1)(x-1)y}{(x+2)y+9(x+2)(x+1)} \\
&= \frac{3xy+3y-5x^2y+5y}{xy+2y+9x^2+27x+18} \\
&= \frac{3xy-5x^2y+8y}{9x^2+xy+2y+27x+18}
\end{aligned}$$

To check the solution, we will subsitute $x=2$ and $y=3$ into the original expression and our result. We expect both to evaluate to the same number.

$$\frac{\frac{3}{2^2+2-2}-\frac{5}{2+2}}{\frac{1}{2^2-1}+\frac{9}{3\cdot 2-3}} = \frac{\frac{3}{4}-\frac{5}{4}}{\frac{1}{3}+\frac{9}{3}} = \frac{-\frac{2}{4}}{\frac{10}{3}} = -\frac{1}{2}\,\frac{3}{10} = -\frac{3}{20}$$

$$\frac{3\cdot 2\cdot 3-5\cdot 2^2\cdot 3+8\cdot 3}{9\cdot 2^2+2\cdot 3+2\cdot 3+27\cdot 2+18} = \frac{18-60+24}{36+6+6+54+18} = \frac{-18}{120} = -\frac{3}{20} \qquad \checkmark$$

13. (10 pts) Find all real number solutions for x of the equation

$$\frac{2}{x^2-5x+6}-\frac{1}{x-3}=\frac{7}{x^2+x-12}.$$

Check your solutions.

$$\begin{aligned}
\frac{2}{x^2-5x+6}-\frac{1}{x-3} &= \frac{7}{x^2+x-12} \\
\frac{2}{x^2-5x+6}-\frac{1}{x-3}-\frac{7}{x^2+x-12} &= 0 \\
\frac{2}{(x-2)(x-3)}-\frac{1}{x-3}-\frac{7}{(x-3)(x+4)} &= 0 \\
\frac{2(x+4)}{(x-2)(x-3)(x+4)}-\frac{(x-2)(x+4)}{(x-3)(x-2)(x+4)}-\frac{7(x-2)}{(x-3)(x+4)(x-2)} &= 0
\end{aligned}$$

$$
\begin{aligned}
\frac{2(x+4)-(x-2)(x+4)-7(x-2)}{(x-2)(x-3)(x+4)} &= 0\\
\frac{2x+8-(x^2+2x-8)-7x+14}{(x-2)(x-3)(x+4)} &= 0\\
\frac{2x+8-x^2-2x+8-7x+14}{(x-2)(x-3)(x+4)} &= 0\\
\frac{-x^2-7x+30}{(x-2)(x-3)(x+4)} &= 0\\
-\frac{x^2+7x-30}{(x-2)(x-3)(x+4)} &= 0
\end{aligned}
$$

A fraction is 0 if its numerator is 0 and its denominator is not 0. So we want

$$
\begin{aligned}
x^2+7x-30 &= 0\\
(x-2)(x-3)(x+4) &\neq 0
\end{aligned}
$$

To solve the first equation factor the polynomial:

$$x^2+7x-30=(x-3)(x+10)=0$$

so $x=3$ or $x=-10$. But $x=3$ makes the denominator 0. So $x=3$ is a false root. Therefore the only solution is $x=-10$.

Check:

$$\frac{2}{(-10)^2-5\cdot(-10)+6}-\frac{1}{-10-3}=\frac{2}{156}-\frac{1}{-13}=\frac{1}{78}+\frac{1}{13}=\frac{1}{78}+\frac{6}{78}=\frac{7}{78}$$

$$\frac{7}{(-10)^2+(-10)-12}=\frac{7}{78}\qquad\checkmark$$

14. (10 pts) Gerardo and Rosa make gorditas de nata at a booth at the Fiestas del Sol. Gerardo fills a box with gorditas in 12 minutes. When working together with Rosa, they fill a box in 4 minutes. How long does Rosa take to fill a box by herself?

Let x be the time in minutes Rosa takes to fill the box by herself. Then in 1 minute, Gerardo fills $1/12$ of a box, Rosa fills $1/x$ of a box, and the two of them together fill $1/12+1/x$ of a box. But we know that together they fill $1/4$ of a box in a minute. So the following equation must hold

$$
\begin{aligned}
\frac{1}{12}+\frac{1}{x} &= \frac{1}{4} && \text{multiply by } 12x\\
12x\left(\frac{1}{12}+\frac{1}{x}\right) &= 12x\,\frac{1}{4}\\
x+12 &= 3x && \text{subtract } x\\
12 &= 2x && \text{divide by 2}\\
6 &= x
\end{aligned}
$$

So Rosa takes 6 minutes to fill a box with gorditas de nata by herself.

Check: Let's go back to the original problem. If Gerardo takes 12 minutes to fill a box and Rosa takes 6 minutes, then in 4 minutes Gerardo fills $4/12=1/3$ of a box and Rosa fills $4/6=2/3$ of a box. That is $1/3+2/3=1$ box for the two of them combined, which is consistent with what the problem stated.

15. (10 pts) **Extra credit problem.** Let $P(x)/Q(x)$ be a rational expression whose restricted values are $x = -3$, $x = 0$, and $x = 10$. What can you say about the restricted values of the rational expression $P(1/y)/Q(1/y)$, that is the rational expression you get if you replace each x by $1/y$? (Hint: You may want to start answering this question by looking at an example or two of such rational expressions.)

The restricted values of a rational expression are those values that make the denominator 0. That is $Q(-3) = 0$, $Q(0) = 0$, and $Q(10) = 0$.We are told these are all of the restricted values, so $Q(x)$ cannot be 0 for any other value of x.

Now, the restricted values of $P(1/y)/Q(1/y)$ are those values of y for which this rational expression does not have a value. It should be clear that $y = 0$ is one of them because $1/0$ is undefined, so

$$\frac{P\left(\frac{1}{0}\right)}{Q\left(\frac{1}{0}\right)}$$

is also undefined. Another restricted value is $y = -1/3$ because

$$Q\left(\frac{1}{-1/3}\right) = Q(-3) = 0$$

so

$$\frac{P\left(\frac{1}{-1/3}\right)}{Q\left(\frac{1}{-1/3}\right)} = \frac{P(-3)}{Q(-3)} = \frac{P(-3)}{0}$$

For the same reason, $y = 1/10$ is also a restricted value of $P(1/y)/Q(1/y)$. These are all. Since $Q(x) \neq 0$ for any number other than $x = -3$, $x = 0$, and $x = 10$, therefore $Q(1/y) \neq 0$ for any number other than $y = -1/3$ and $y = 1/10$. There is obviously no value of y that would make $1/y = 0$.

So the restricted values of $P(1/y)/Q(1/y)$ are $y = -1/3$, $y = 0$, and $y = 1/10$.