GMS 91 Exam 2 Solutions

Nov 2, 2009

1. (5 pts) If q varies jointly as t and r and inversely as p, then find an equation for q if $q = 6$ when $t = 2$, $r = 8$, and $p = 7$.

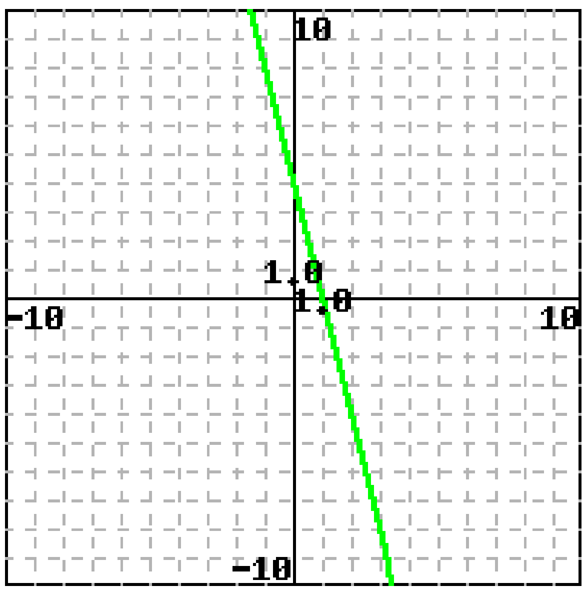
   We know $q = k\frac{tr}{p}$ for some constant k. To find k, we substitute $q = 6$, $t = 2$, $r = 8$, and $p = 7$:

$$6 = k\frac{(-2)(-8)}{7} \implies k = \frac{6 \cdot 7}{(-2)(-8)} = \frac{21}{8}.$$

   Therefore

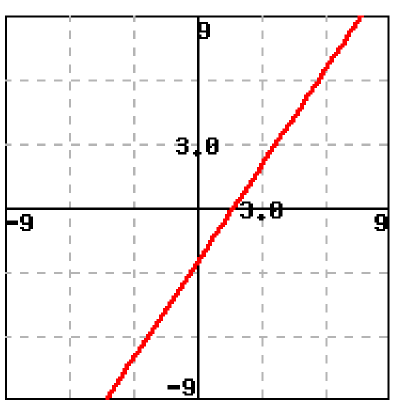
$$q = \frac{21}{8}\,\frac{tr}{p}.$$

2. (10 pts) Find an equation $y = mx + b$ for the line whose graph is sketched below.

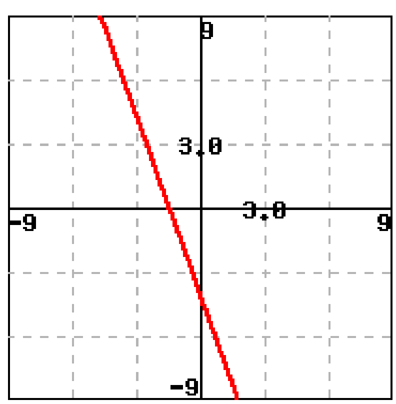


Notice that the line passes through $(0, 4)$ and $(1, 0)$. Therefore its slope is $m = \frac{0-4}{1-0} = -4$. The y-intercept is 4. So the slope-intercept equation is $y = -4x + 4$.

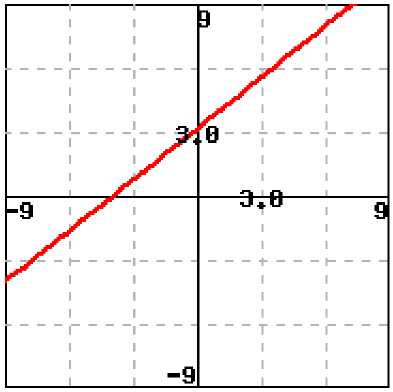
3. (5 pts) Order the following graphs by increasing slope of the line being displayed:



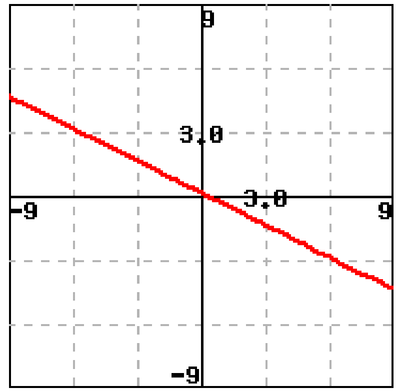
A



B



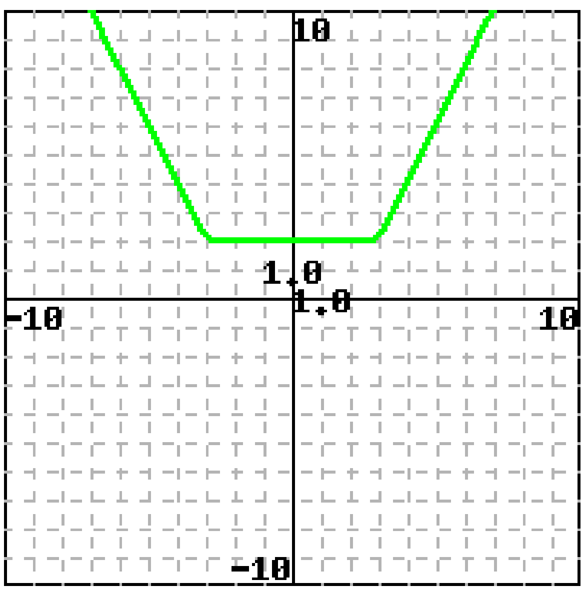
C



D

B is the steepest decreasing line, so it has the least slope. D is also decreasing, so it has a negative slope, but it is not as steep as B. A and C are increasing, so they have positive slopes. A is steeper. Hence the correct order is B, D, C, A.

4. (5 pts) Find the interval over which the function is increasing and the interval over which it is decreasing.



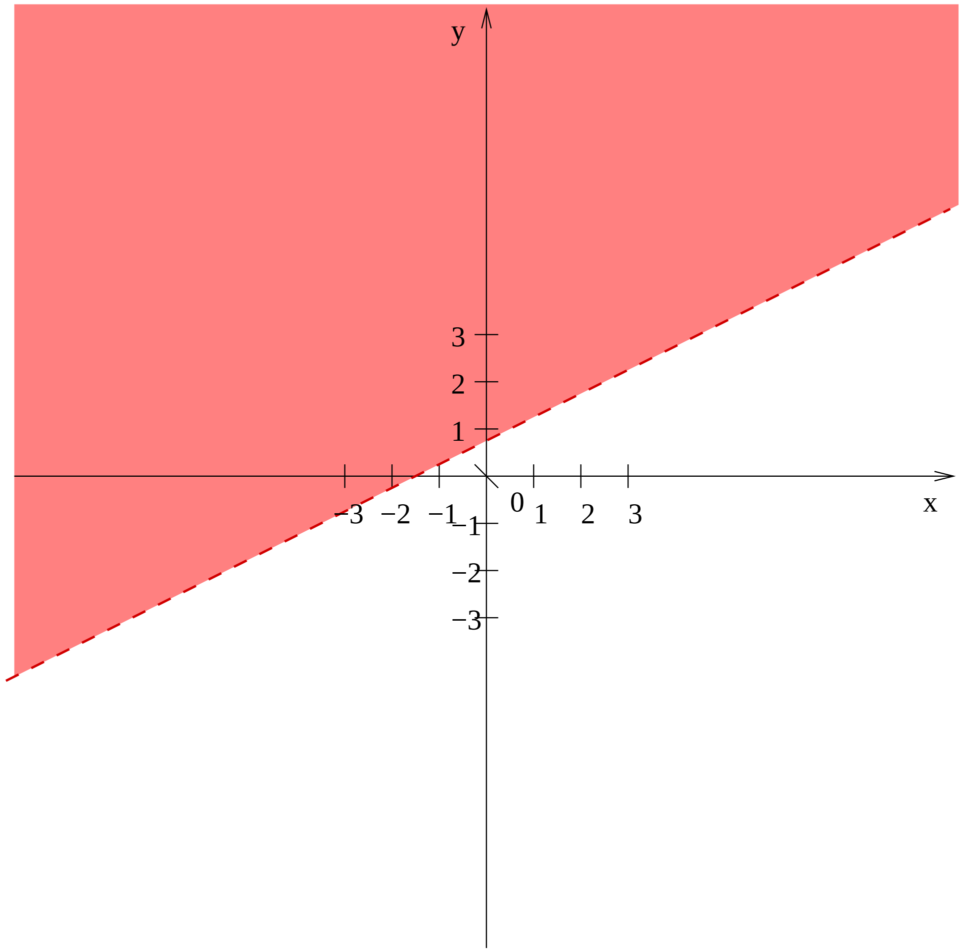
The function is decreasing over $(-\infty, 3]$ and increasing over $[-3, \infty)$. It is strictly decreasing (our textbook's definition of decreasing) over $(-\infty, -3]$ and strictly increasing (our textbook's definition of increasing) over $[3, \infty)$.

5. (10 pts) Graph the solution set of the inequality $2x < 4y - 3$. Be sure to indicate if the boundary of the region is included or not. Explain how you know which part of the plain satisfies the inequality.

First we plot the line

$$2x = 4y - 3 \implies 4y = 2x + 3 \implies y = \frac{1}{2}x + \frac{3}{4}.$$

Imagine you are standing at a point on this line. At that point $2x = 4y - 3$. If you move to the left, then x decreases and y does not change, so $2x < 4y - 3$ there. Therefore the solution set of $2x < 4y - 3$ is the half-plain left of the line. Since the inequality is strict, the line itself is not part of the solution set.

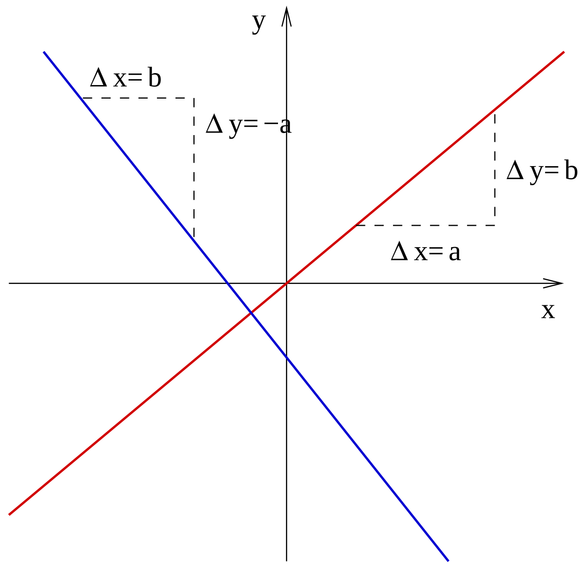


6. (a) (10 pts) Suppose that the lines $y = m_1x + b_1$ and $y = m_2x + b_2$ are perpendicular. What can you say about their slopes? Why is this so?

We can say $m_1m_2 = -1$. This is because the two right triangles below are congruent. So if $m_1 = b/a$, then $m_2 = -a/b$. Hence

$$m_1m_2 = \frac{b}{a}\frac{-a}{b} = -1.$$

A note of proper terminology: You cannot say that m_1 and m_2 are opposites. This would mean $m_2 = -m_1$. Words like opposite, reciprocal, and negative have specific meanings in math and they should be used in only those specific meanings. The proper use of terminology is essential is academia and in the professions. Imagine the confusion that could arise if your doctor (pilot, pharmacist, you name it) misused terminology when communicating on the job.



(b) (10 pts) Find the equation of the line which is perpendicular to $y = 3x - 4$ and passes through the point $(-6, 2)$.

The slope of the original line is 3. So the slope of a perpendicular line is $-1/3$. Now substitute $(-6, 2)$ into $y = 1/3x + b$:

$$2 = -\frac{1}{3}(-6) + b \implies 2 = 2 + b \implies b = 0.$$

So the equation is $y = 2x$.

7. (5 pts) Let a be a real number. Express a^7a^{10} as a single power of a. Explain why you can do this. In particular, what basic principle(s) of algebra do you need to use?

$$a^7a^{10} = (\underbrace{a \cdot a \cdots a}_{7 \text{ times}})(\underbrace{a \cdot a \cdots a}_{10 \text{ times}}) = \underbrace{a \cdot a \cdots a}_{7 \text{ times}}\underbrace{a \cdot a \cdots a}_{10 \text{ times}} = a^{17}$$

where we used the associativity of multiplication to get rid of the parentheses at the second equality.

8. (5 pts each) **Extra credit problem.** Recall the definition of increasing function. Let l be the line whose equation is $y = mx + b$.

(a) Prove that l is the graph of an increasing function if $m \geq 0$.

Suppose $m \geq 0$. Let $f(x) = mx + b$. The line l is the graph of f.
Recall that a function f is called increasing if $f(x_1) \leq f(x_2)$ whenever $x_1 < x_2$. Let x_1 and x_2 be such that $x_1 < x_2$. Multiply this inequality by m to get $mx_1 \leq mx_2$. (The $<$ became $\leq$ because m could be 0.) Add b to this to get $mx_1 + b \leq mx_2 + b$. Therefore $f(x_1) \leq f(x_2)$. This shows f is an increasing function.

(b) Prove that if l is the graph of an increasing function then $m \geq 0$.

Let $f(x) = mx+b$. If f is increasing then for any $x_1 < x_2$, $f(x_1) \leq f(x_2)$. In particular, this is true for $x_1 = 0$ and $x_2 = 1$. So $f(0) \leq f(1)$. But $f(0) = b$ and $f(1) = m + b$, so $b \leq m + b$. Subtract b from both sides to conclude $0 \leq m$.