GMS 91 Exam 1 Solutions

Oct 6, 2010

11. (10 pts) Solve the equation

$$5(2-3x) = 23$$

for the real number x. Justify each step by referring to an appropriate algebraic property. Be sure not to skip or combine steps.

$$\begin{aligned}
5(2-3x) &= 23 && \\
10 + (-15)x &= 23 && \text{distributivity of multiplication over addition} \\
(-10) + (10 + (-15)x) &= (-10) + 23 && \text{addition property of equality} \\
(-10) + (10 + (-15)x) &= 13 && \text{arithmetic} \\
((-10) + 10) + (-15)x &= 13 && \text{associativity of addition} \\
0 + (-15)x &= 13 && \text{inverse property of addition} \\
(-15)x &= 13 && \text{identity property of addition} \\
\left(-\frac{1}{15}\right)((-15)x) &= \left(-\frac{1}{15}\right) 13 && \text{multiplication property of equality} \\
\left(-\frac{1}{15}\right)((-15)x) &= -\frac{13}{15} && \text{arithmetic} \\
\left(\left(-\frac{1}{15}\right)(-15)\right) x &= -\frac{13}{15} && \text{associativity of multiplication} \\
1 \cdot x &= -\frac{13}{15} && \text{inverse property of multiplication} \\
x &= -\frac{13}{15} && \text{identity property of multiplication}
\end{aligned}$$

12. (10 pts)
   (a) Solve the absolute value equation

$$|2x+3| = 6 + |5-3x|$$

for x, where x is a real number. You do not have to justify each step, but be sure to find all solutions.

We have four cases since each absolute value means either leaves its argument alone, or reverses its sign.

(i) Case $|2x+3| = 2x+3$ and $|5-3x| = 5-3x$.

$$\begin{aligned}
2x+3 &= 6 + (5-3x) \\
2x+3 &= 11 - 3x \\
5x &= 8 \\
x &= \frac{8}{5}
\end{aligned}$$

But this could be a false root, so we need to check it.

$$\left|2\,\frac{8}{5}+3\right| = \left|\frac{16}{5}+\frac{15}{5}\right| = \left|\frac{31}{5}\right| = \frac{31}{5}$$

$$6+\left|5-3\,\frac{8}{5}\right| = 6+\left|\frac{25}{5}-\frac{24}{5}\right| = 6+\left|\frac{1}{5}\right| = 6+\frac{1}{5} = \frac{30}{5}+\frac{1}{5} = \frac{31}{5}$$

So this is indeed a solution.

(ii) Case $|2x+3| = -(2x+3)$ and $|5-3x| = 5-3x$.

$$\begin{aligned} -(2x+3) &= 6+(5-3x) \\ -2x-3 &= 11-3x \\ x &= 14 \end{aligned}$$

But this could be a false root, so we need to check it.

$$|2(14)+3| = |28+3| = |31| = 31$$

$$6+|5-3(14)| = 6+|5-42| = 6+|-37| = 6+37 = 43$$

Aha, this is a false root!

(iii) Case $|2x+3| = 2x+3$ and $|5-3x| = -(5-3x)$.

$$\begin{aligned} 2x+3 &= 6-(5-3x) \\ 2x+3 &= 6-5+3x \\ 2x+3 &= 1+3x \\ 2 &= x \end{aligned}$$

But this could be a false root, so we need to check it.

$$|2(2)+3| = |4+3| = |7| = 7$$

$$6+|5-3(2)| = 6+|5-6| = 6+|-1| = 6+1 = 7$$

So this is indeed a solution.

(iv) Case $|2x+3| = -(2x+3)$ and $|5-3x| = -(5-3x)$.

$$\begin{aligned} -(2x+3) &= 6-(5-3x) \\ -2x-3 &= 6-5+3x \\ -2x-3 &= 1+3x \\ -5x &= 4 \\ x &= -\frac{4}{5} \end{aligned}$$

But this could be a false root, so we need to check it.

$$\left|2\left(-\frac{4}{5}\right)+3\right| = \left|-\frac{8}{5}+\frac{15}{5}\right| = \left|\frac{7}{5}\right| = \frac{7}{5}$$

$$6+\left|5-3\left(-\frac{4}{5}\right)\right| = 6+\left|\frac{25}{5}+\frac{12}{5}\right| = 6+\left|\frac{37}{5}\right| = 6+\frac{37}{5} = \frac{30}{5}+\frac{37}{5} = \frac{67}{5}$$

Another false root.

(b) Check your solutions to the equation in part (a) if you have not already done so.

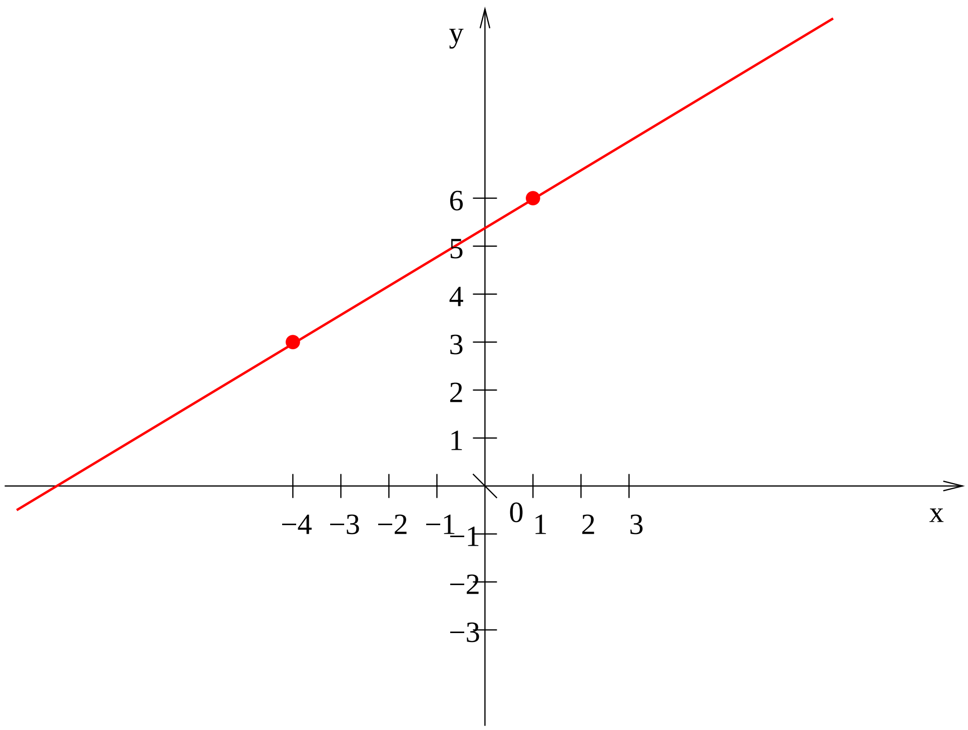
I have already done that while I checked which roots were really roots.

13. (10 pts) Let l be the line with equation

$$y - 3 = \frac{3}{5}(x + 4).$$

(a) Graph l.

This is a point-slope equation of a line. It immediately shows that l passes through $(-4, 3)$ and has a slop of $3/5$. The latter means it also passes through $(-4+5, 3+3) = (1, 6)$. So here is the graph:



(b) Find the slope-intercept equation of the line which is parallel to l and passes through the point $(1, 8)$.

The slope of l is $3/5$. Any line that is parallel to l will have the same slope. So the equation of the line we are looking for is $y = 3/5x + b$ where b is the y-intercept. We can determine b by subtituting the point $(1, 8)$ into this equation.

$$8 = \frac{3}{5}(1) + b$$
$$b = 8 - \frac{3}{5} = \frac{40}{5} - \frac{3}{5} = \frac{37}{5}$$

So the equation of the line is

$$y = \frac{3}{5}x + \frac{37}{5}.$$

14. (10 pts) **Extra credit problem.** Show that if x is any real number, then $0 \cdot x$ must be 0. In your argument, you may use any of the standard algebraic properties of the real numbers, except of course for the zero product rule itself, since that is what you are trying to prove.

We did this via the following argument. Let $y = 0 \cdot x$. Whatever number y is,

$$\begin{aligned} y + y &= 0 \cdot x + 0 \cdot x \\ &= (0+0)x && \text{by distributivity of multiplication over addition} \\ &= 0 \cdot x && \text{because 0 is the additive identity} \\ &= y \end{aligned}$$

So

$$\begin{aligned} y + y &= y \\ -y + (y + y) &= -y + y && \text{by the addition property of equality} \\ (-y + y) + y &= -y + y && \text{by associativity of addition} \\ 0 + y &= 0 && \text{by additive inverses} \\ y &= 0 && \text{because 0 is the additive identity} \end{aligned}$$

So $0 \cdot x = y = 0$.