GMS 91 Exam 1 Solutions

Feb 16, 2011

See solutions to problems 1–12 in Hawkes Learning System.

13. (10 pts) Give a mathematical argument why $(-3)(-6) = 18$. (Hint: Saying that your elementary school math teacher told you that negative times negative is not a mathematical argument.)

    First, we will look at $3(-6)$. Multiplication by a positive integer is repeated addition, so

    $$3(-6) = (-6) + (-6) + (-6) = -18.$$

    Now

    $$0 = 0 \cdot (-6) = (-3 + 3)(-6) = (-3)(-6) + 3(-6) = (-3)(-6) - 18$$

    by the distributive law. Add 18 to both sides of this equality to get

    $$18 = (-3)(-6) - 18 + 18 = (-3)(-6).$$

14. (10 pts) Let x be any integer. Prove that $0 \cdot x = 0$.

    Let $y = 0 \cdot x$. Then

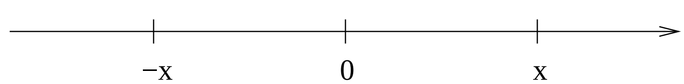
    $$y = 0 \cdot x = (0 + 0) \cdot x = 0 \cdot x + 0 \cdot x = y + y$$

    where we used $0 = 0 + 0$ and the distributive law. Therefore $y = y + y$. Now subtract y from both sides to get $0 = y = 0 \cdot x$.

    Note that saying $0 \cdot x = 0$ is so because of the Zero Factor Law is circular reasoning. In fact, $0 \cdot x = 0$ for all numbers x is the Zero Factor Law. Circular reasoning is when you try to prove something and in your proof you assume you already know that same thing is true. Circular reasoning is not an accepted form of logical justification.

15. (10 pts) Find all real numbers x for which $|-x| = x$. Give your solution either in set builder notation or interval notation. Be sure to fully justify your work.

    The absolute value of $-x$ is the distance of $-x$ from 0. Notice that $-x$ is always the same distance from 0 as x. This is because x and $-x$ are in symmetric positions with respect to 0 on the number line:

    So $|-x| = |x|$ for every real number x. Now we can rephrase the question: for which real numbers x is $|x| = x$. But we already know the answer to that one. By the definition of the absolute value $|x| = x$ for all number $x \geq 0$. On the other hand, when $x < 0$, $|x|$ cannot be equal to x, since $|x|$ is never negative. Therefore the answer is

    $$\{x \mid x \geq 0\} = [0, \infty).$$

16. (5 pts each) **Extra credit problem.** A unit fraction is a rational number with 1 in the numerator. E.g. $1/2$ and $1/10$ are unit fractions, but $2/3$ is not. The ancient Egyptians wrote all positive rational numbers as a sum of whole number and some unit fractions. E.g. $14/3$ was $4 + 1/2 + 1/6$. They would not repeat the same unit fraction more than once. E.g. they would not write $2/3$ as $1/3 + 1/3$, but as $1/2 + 1/6$.

(a) Use the Egyptian method to describe the number 59/24.

Since 59/24 is between 2 and 3, we can use 2 as the whole number. Now notice

$$\frac{59}{24} = 2 + \frac{11}{24} = 2 + \frac{8+3}{24} = 2 + \frac{8}{24} + \frac{3}{24} = 2 + \frac{1}{3} + \frac{1}{8}.$$

(b) Find a different combination of whole numbers and unit fractions to write 59/24.

There are many ways to do this. One is

$$\frac{59}{24} = 2 + \frac{11}{24} = 2 + \frac{6+4+3}{24} = 2 + \frac{6}{24} + \frac{4}{24} + \frac{3}{24} = 2 + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}.$$