GMS 91 Exam 2 Solutions

Mar 21, 2011

See solutions to problems 1–13 in Hawkes Learning System.

14. (10 pts) Solve the equation

$$4x + 2 = 7 - x$$

for the real number x. Justify each step by referring to an appropriate algebraic property. Be sure not to skip or combine steps.

$4x + 2 = 7 - x$	
$4x + 2 = 7 + (-x)$	definition of subtraction
$(4x + 2) + x = (7 + (-x)) + x$	addition property of equality
$(4x + 2) + x = 7 + ((-x) + x)$	associativity of addition
$(4x + 2) + x = 7 + 0$	inverse property of addition
$(4x + 2) + x = 7$	identity property of addition
$(2 + 4x) + x = 7$	commutativity of addition
$2 + (4x + x) = 7$	associativity of addition
$2 + (4 + 1)x = 7$	distributivity of multiplication over addition
$2 + 5x = 7$	arithmetic
$(-2) + (2 + 5x) = (-2) + 7$	addition property of equality
$((-2) + 2) + 5x = (-2) + 7$	associativity of addition
$0 + 5x = (-2) + 7$	inverse property of addition
$5x = (-2) + 7$	identity property of addition
$5x = 5$	arithmetic
$\frac{1}{5}(5x) = \frac{1}{5}5$	multiplication property of equality
$\left(\frac{1}{5}5\right)x = \frac{1}{5}5$	associativity of multiplication
$1x = 1$	inverse property of multiplication
$x = 1$	identity property of multiplication

15. (12 pts) Solve the absolute value inequality

$$|5 + x| - 12 \leq |x + 1|$$

where x is a real number. Express your final answer in interval notation. You do not have to justify each step, but be sure to find all solutions. (Hint: be sure to verify that your solutions are indeed solutions.)

We have four cases since each absolute value either leaves its argument alone, or reverses its sign.

Case $|5 + x| = 5 + x$ and $|x + 1| = x + 1$: This happens when $5 + x \geq 0$ and $x + 1 \geq 0$.

$$\begin{aligned}(5+x)-12 &\leq x+1\\ x-7 &\leq x+1\\ -7 &\leq 1\end{aligned}$$

Actually, this is always true regardless of the value of x. So any real number x satisfies the inequality as long as we are in this case. To be in this case $5+x \geq 0$, so $x \geq -5$ and $x+1 \geq 1$, so $x \geq -1$. Of this $x \geq -1$ is the stronger condition. So this case yields $x \geq -1$ as a set of solutions.

Case $|5+x| = -(5+x)$ and $|x+1| = x+1$: This happens when $5+x < 0$ and $x+1 \geq 0$. Before we start to compute away, notice that $5+x < 0$ implies $x < -5$ and $x+1 \geq 0$ implies $x \geq -1$. But there is no real number that satisfies both of these conditions. So there are no solutions in this case.

Case $|5+x| = 5+x$ and $|x+1| = -(x+1)$: This happens when $5+x \geq 0$ and $x+1 < 0$.

$$\begin{aligned}(5+x)-12 &\leq -(x+1)\\ x-7 &\leq -x-1\\ 2x &\leq 6\\ x &\leq 3\end{aligned}$$

But x also has to satisfy $5+x \geq 0$, which implies $x \geq -5$, and $x+1 < 0$, which implies $x < -1$. The latter is a stronger condition than $x \leq 3$. So $-5 \leq x < -1$.

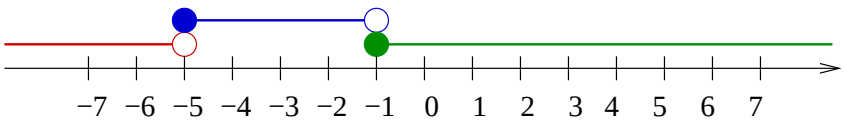
Case $|5+x| = -(5+x)$ and $|x+1| = -(x+1)$: This happens when $5+x < 0$ and $x+1 < 0$.

$$\begin{aligned}-(5+x)-12 &\leq -(x+1)\\ -5-x-12 &\leq -x-1\\ -x-17 &\leq -x-1\\ -17 &\leq -1\end{aligned}$$

Actually, this is always true regardless of the value of x. So any real number x satisfies the inequality as long as we are in this case. To be in this case $5+x < 0$, so $x < -5$ and $x+1 < 1$, so $x < -1$. Of this $x < -5$ is the stronger condition. So this case yields $x < -5$ as a set of solutions.

Combining these results, x is a solution of this inequality if $x \geq -1$ or $-5 \leq x < -1$ or $x < -5$. This means

$$x \in (-\infty, -5) \cup [-5, -1) \cup [-1, \infty) = (-\infty, \infty) = \mathbb{R}$$



16. (a) (3 pts) What is a relation? State the definition.

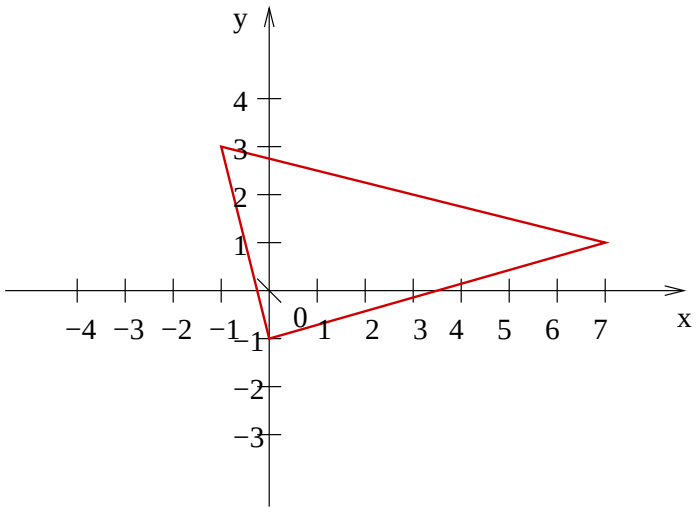
A relation is a set of ordered pairs.

(b) (5 pts) Give an example of a relation. What is the domain of your relation? What is the range?

One such example is when you pair your friends with the make and model of vehicles they own. It may look something like this:

$$\{(\text{Christian, Toyota Tundra}), (\text{Olav, Mazda Protege}), (\text{Ivan, Mercedes C-class}),$$
$$(\text{Ivan, Mercedes G-class}), (\text{Jecsi, Nissan Versa}), (\text{Gerardo, Ford Ranger})\}$$

17. (10 pts) **Extra credit problem.** The three points $(-1, 3)$, $(0, -1)$, and $(7, 1)$ form a triangle in the plane. How could you decide with certainty whether this triangle is a right triangle? Is it?



First, plotting the points suggests that the angle at $(0, -1)$ may be a right angle, but no matter how accurate your plot is, it does not tell you with certainty. That angle could be just a little different from a right angle.

One way to be certain, is to look at the slope of the three sides. These are

$$m_1 = \frac{3-(-1)}{-1-0} = \frac{4}{-1} = -4$$
$$m_2 = \frac{1-(-1)}{7-0} = \frac{2}{7}$$
$$m_3 = \frac{1-3}{7-(-1)} = \frac{-2}{8} = -\frac{1}{4}$$

For two of these to be perpendicular, their product must be -1.

$$m_1 m_2 = (-4)\,\frac{2}{7} = -\frac{8}{7} \neq -1$$
$$m_2 m_3 = \frac{2}{7}\,\frac{1}{4} \neq -1$$
$$m_3 m_1 = -\frac{1}{4}\,(-4) = 1 \neq -1$$

So this triangle does not have a right angle. The angle at $(0, -1)$ which looks like it could be one, is not. If you are curious, it is about $88.1°$.