# GMS 91 Exam 1 Solutions

Feb 22, 2012

See solutions to problems 1–13 in Hawkes Learning System.

15. (15 pts) Use cases to solve the absolute value inequality

$$|4x - 5| > 2 + |3 - x|$$

where $x$ is a real number. Express your final answer in interval notation.

We have four cases since each absolute value either leaves its argument alone, or reverses its sign.

1. $4x - 5 \geq 0$ and $3 - x \geq 0$. This happens when

$$4x \geq 5 \iff x \geq \frac{5}{4}$$

and

$$3 \geq x.$$

If so, $|4x - 5| = 4x - 5$ and $|3 - x| = 3 - x$. So the inequality is

$$\begin{aligned} 4x - 5 &> 2 + 3 - x \\ 4x - 5 &> 5 - x \\ 5x &> 10 \\ x &> 2. \end{aligned}$$

So this case yields the solution set $x > 2$ and $x \geq 5/4$ and $3 > x$. Notice that whenever $x > 2$, it is also true that $x \geq 5/4$. So we get $2 < x \leq 3$ or $x \in (2, 3]$.

2. $4x - 5 \geq 0$ and $3 - x < 0$. This happens when $x \geq 5/4$ and $3 < x$. If so, $|4x - 5| = 4x - 5$ and $|3 - x| = -(3 - x)$. So the inequality is

$$\begin{aligned} 4x - 5 &> 2 - (3 - x) \\ 4x - 5 &> x - 1 \\ 3x &> 4 \\ x &> \frac{4}{3}. \end{aligned}$$

So this case yields the solution set $x > 4/3$ and $x \geq 5/4$ and $3 < x$. Notice that whenever $x > 3$, it is also true that $x > 4/3$ and $x \geq 5/4$. So we get $3 < x$ or $x \in (3, \infty)$.

3. $4x - 5 < 0$ and $3 - x \geq 0$. This happens when $x < 5/4$ and $3 \geq x$. If so, $|4x - 5| = -(4x - 5)$ and $|3 - x| = 3 - x$. So the inequality is

$$\begin{aligned} -(4x - 5) &> 2 + 3 - x \\ 5 - 4x &> 5 - x \\ 0 &> 3x \\ 0 &> x. \end{aligned}$$

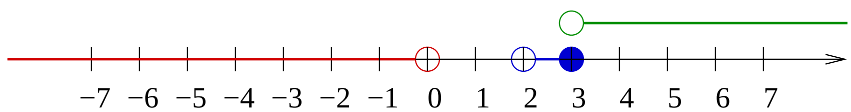
So this case yields the solution set $x < 0$ and $x < 5/4$ and $3 \geq x$. Notice that whenever $x < 0$, it is also true that $x < 5/4$ and $x \leq 3$. So we get $x < 0$ or $x \in (-\infty, 0)$.

4. $4x - 5 < 0$ and $3 - x < 0$. This happens when $x < 5/4$ and $3 < x$. If so, $|4x - 5| = -(4x - 5)$ and $|3 - x| = -(3 - x)$. So the inequality is

$$\begin{aligned} -(4x-5) &> 2 - (3-x) \\ 5 - 4x &> x - 1 \\ 6 &> 5x \\ \frac{6}{5} &> x. \end{aligned}$$

So this case yields the solution set $x < 6/5$ and $x < 5/4$ and $3 > x$. Notice that this is the empty set, since no number $x$ can be both more than 3 and less than $5/4$.

I will combine these results on the number line:




That is $x < 0$ or $x > 2$. Or in interval notation $x \in (-\infty, 0) \cup (2, \infty)$

16. (a) (3 pts) Define what the slope of a line is.

The slope of a line is the rise divided by the run between two points $(x_1, y_1)$ and $(x_2, y_2)$ on the line, where the rise is the difference in the vertical coordinates $\Delta y = y_2 - y_1$ and the run is the difference in the horizontal coordinates $\Delta x = x_2 - x_1$.

(b) (2 pts) Explain in your own words what a slope of $-2/5$ tells you about the steepness of a line.

This means that if you move 5 units right on the line then you need to move 2 units down. Or if you move 5 units left, you need to move 2 units up.

(c) (10 pts) Find the slope-intercept equation of the line which passes through the point $(-2, -4)$ and is perpendicular to the line $2x + 3y = 12$.

$$\begin{aligned} 2x + 3y &= 12 \\ 3y &= -2x + 12 \\ y &= -\frac{2}{3}x + 4. \end{aligned}$$

So the slope of the line $2x + 3y = 12$ is $-2/3$. The slope of a perpendicular line is then $3/2$. So the equation of the perpendicular line through $(-2, -4)$ is

$$\begin{aligned} y - (-4) &= \frac{3}{2}(x - (-2)) \\ y + 4 &= \frac{3}{2}(x + 2) \\ y &= \frac{3}{2}x + 3 - 4 = \frac{3}{2}x - 1. \end{aligned}$$

17. (10 pts) **Extra credit problem.** Let $l_1$, $l_2$, and $l_3$ be three distinct lines in the Cartesian plane. Use your knowledge of slope to prove that if $l_1$ and $l_2$ are both perpendicular to $l_3$, then $l_1$ must be parallel to $l_2$. (Hint: to have a complete argument, you will have to consider that some of the lines could be vertical too.)

Let $l_1$ and $l_2$ be perpendicular to $l_3$. We need to consider three cases:

1. If $l_3$ is horizontal, then $l_1$ and $l_2$ are both vertical, hence they are parallel.
2. If $l_3$ is vertical, then $l_1$ and $l_2$ are both horizontal, hence they are parallel.
3. If $l_3$ is neither horizontal nor vertical, then $l_3$ has a slope $m_3$ and $m_3 \neq 0$. Since $l_1$ is perpendicular to $l_3$, its slope $m_1$ is $-1/m_3$. Since $l_2$ is perpendicular to $l_3$, its slope $m_2$ is $-1/m_3$. Now $m_1 = m_2$, so $l_1$ is parallel to $l_2$.