GMS 91 Exam 2 Solutions

Apr 11, 2012

See solutions to problems 1–12 in Hawkes Learning System.

13. (5 pts) Find the largest possible domain in the set of real numbers that makes the formula

$$f(x) = \frac{x}{2 - |x|}$$

a function.

The only thing to worry about is that the denominator cannot be 0. This would happen if $|x| = 2$, which is when $x = 2$ or $x = -2$. So the largest possible domain is all real numbers except 2 and -2.

$$D(f) = \{x \in \mathbb{R} \mid x \neq \pm 2\} = (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$$

14. (5 pts each)

(a) Explain <u>why</u>

$$7^4 7^5 = 7^9.$$

$$\begin{aligned} 7^4 7^5 &= (7 \cdot 7 \cdot 7 \cdot 7)(7 \cdot 7 \cdot 7 \cdot 7 \cdot 7) \\ &= 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 && \text{by the associativity of multiplication} \\ &= 7^9 \end{aligned}$$

(b) Let a be any real number and m, n positive integers. Show that

$$a^m a^n = a^{m+n}.$$

$$\begin{aligned} a^m a^n &= (\underbrace{a \cdot a \cdots a}_{m \text{ factors}})(\underbrace{a \cdot a \cdots a}_{n \text{ factors}}) \\ &= \underbrace{a \cdot a \cdots a}_{m+n \text{ factors}} && \text{by the associativity of multiplication} \\ &= a^{m+n} \end{aligned}$$

15. (a) (10 pts) Evaluate

$$\frac{6y^3 + 13y^2 - 6}{2y + 3}$$

using the long-hand polynomial division algorithm. Be sure to show all the details of your computation.

$$
\begin{array}{r}
3y^2 + 2y - 3 \\
2y+3 \overline{\big)\; 6y^3 + 13y^2 \qquad - 6} \\
\underline{-6y^3 - 9y^2 \qquad\qquad} \\
4y^2 \qquad\quad \\
\underline{-4y^2 - 6y \quad} \\
-6y - 6 \\
\underline{6y + 9} \\
3
\end{array}
$$

So the result is

$$(6y^3 + 13y^2 - 6)/(2y + 3) = 3y^2 + 2y - 3 + \frac{3}{2y + 3}$$

or

$$(6y^3 + 13y^2 - 6) = (3y^2 + 2y - 3)(2y + 3) + 3.$$

(b) (5 pts) Check your answer to part (a).

$$\begin{aligned}(3y^2 + 2y - 3)(2y + 3) + 3 &= 6y^3 + 9y^2 + 4y^2 + 6y - 6y - 9 + 3 \\ &= 6y^3 + 13y^2 - 6\end{aligned}$$

16. (10 pts) **Extra credit problem.** Let $p(x)$ and $q(x)$ be two polynomials whose coefficients are real numbers. Prove that the product $p(x)q(x)$ is always a polynomial with real coefficients. (Hint: try an example or two for intuition, but keep in mind that an example does not prove that the statement is always true.)

First, notice that if you multiply two monomials ax^m and bx^n with real coefficients a and b, you get abx^{m+n}. Since ab is again a real number and $m+n$ is again a nonnegative integer, abx^{m+n} is again a monomial with a real coefficient. Now, $p(x)$ and $q(x)$ are sums of such monomials. When you multiply them together using the distributive property, the result will be a sum of monomials with real coefficients. So $p(x)q(x)$ is again a polynomial with real coefficients.