Name: ______________________

GMS 91 Homework on powers and exponents

Mar 12, 2012

Since this is homework, you should complete this worksheet individually. As usual with homework, I don't mind if you share ideas with each other, but you copying each other's work is cheating. So don't!

We have talked about powers and exponents in class. After some thinking about the meaning of powers, we gave the following definition.

Definition. Let a be any nonzero real number and n any integer. Then the n-th power of a is

$$a^n = \begin{cases} \underbrace{a \cdot a \cdots a}_{n \text{ factors}} & \text{if } n > 0 \\ 1 & \text{if } n = 0 \\ \frac{1}{a^{-n}} & \text{if } n < 0 \end{cases}.$$

For $n > 0$, $0^n = 0$. For $n \leq 0$, 0^n is undefined.

For example, let's use this definition to figure out what 2^{-3} is. First, $-3 < 0$, so we are in the last case, which says $2^{-3} = 1/2^3$. Now $3 > 0$, so $2^3 = 2 \cdot 2 \cdot 2 = 8$. Hence $2^{-3} = 1/8$.

You will now prove the Product Rule of exponents. Since the definition of powers breaks down into cases depending on the sign of the exponent, we will have to consider several cases as you construct your argument.

1. First, let a be a nonzero real number and $m, n > 0$. Show that
$$a^m a^n = a^{m+n}.$$

2. Now, show that if $m, n > 0$ then
$$0^m 0^n = 0^{m+n}.$$

3. Now, let a be a nonzero real number, m any integer, and $n = 0$. Show that
$$a^m a^n = a^{m+n}.$$

4. Let a be a nonzero real number, $m = 0$, and n any integer. Show that
$$a^m a^n = a^{m+n}.$$

5. Let a be a nonzero real number and $m, n < 0$. Show that
$$a^m a^n = a^{m+n}.$$

   If you have a hard time picturing m and n being negative numbers, try a specific example first, say $m = -3$, $n = -2$.

6. Here comes the trickiest case. Let a be a nonzero real number and $m > 0$ and $n < 0$. Show that
$$a^m a^n = a^{m+n}.$$
To do this, you will probably want to consider three cases: $m > -n$, $m = -n$, and $m < -n$. Looking at a few specific examples first may be helpful.

7. Let a be a nonzero real number and $m < 0$ and $n > 0$. Explain how this case follows from the previous one by using the commutative property of multiplication.

We are now ready to conclude that $a^m a^n = a^{m+n}$ in all cases when a^m and a^n are defined.