Math 1101 Practice Final

Dec 18, 2005

Unless told otherwise, your answers must be carefully justified. It is not enough to have a correct answer, you must explain how you got it. Neat work, clear and to-the-point explanations will receive more credit than messy, chaotic answers. You will not be allowed to use books, notes, and calculators on the final exam. Simplify your answers as much as you can. It's ok to leave things like $\ln 2 - \sqrt{3}$ and $\pi/3$ in your answers, but not $\log 20 - \log 2$ or $\sin(\pi/2)$.

1. (a) Let f be an even differentiable function. Prove or give a counterexample: f' is and odd function.

   (b) True or false: If f is continous at a, then $\lim\limits_{x \to a^+} f(x) = \lim\limits_{x \to a^-} f(x)$? Justify your answer.

   (c) True or false: If $f'(a) = 0$ and $f''(a) = 0$, then f has neither a local minimum nor a local maximum at a? Justify your answer.

   (d) Let f be continuous on $[a, b]$ and suppose that f has an absolute minimum at $c \in [a, b]$ and f. Must f also have a local minimum at c? If so, prove it, if not, give a counterexample.

   (e) True or false: If f and g are both differentiable and g is not 0 in some open interval around a, then
   $$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}?$$
   Justify your answer.

   (f) Let f be a differentiable function whose derivative f' is continuous. If $f(1) = -1$, use the Fundamental Theorem of Calculus to find $g(x)$, where
   $$g(x) = \int_x^1 f'(t)\, dt$$

2. Evaluate the following. Be sure to justify your answers.

   (a) $\displaystyle \lim_{x \to \infty} \frac{3 - x^2}{x^4 + 7x^3 - 2}$

   (b) $\displaystyle \lim_{x \to \infty} \left(\frac{1}{x}\right)^x$

   (c) $\displaystyle \frac{d}{dx} x^{\ln x}$

   (d) $\displaystyle \frac{d}{dt} \frac{\sin(\sqrt[3]{1 - t})}{\cos^2 t}$

3. Assume that the function f is differentiable at the number a, that is, $f'(a)$ exists. Use the definition of the derivative to differentiate $f^2(x)$ at $x = a$. (Hint: factor the numerator and use the continuity of f at a to find the limit.)

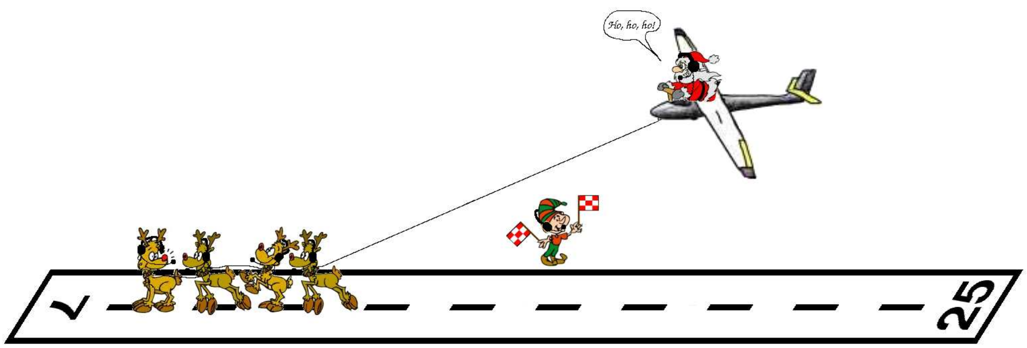
4. Find the equation of the tangent line to the curve $e^{x^2+y^2} = 5$ at the point $(1, \sqrt{\ln 5 - 1})$.

5. Because of new FAA regulations prohibiting flight operations by livestock of foreign registry in the airspace of major US airports, Santa Claus is unable to use his usual means of transportation this year. Having maxed out his credit cards on holiday shopping, all he can afford is a used glider. One essential instrument in a glider is the Vertical Speed Indicator (VSI),

which is the intrument that shows your rate of climb or decent in m/min. Santa is concerned that the VSI in his beat-up glider may be innaccurate, so he decides to calibrate it.

Since thermals are rare in the North pole winter, the glider is launched by having his trusty reindeer tow it down the runway. The length of the towing cable is $l = 500\,\mathrm{m}$.

When Santa's altitude is $400\,\mathrm{m}$, his hand-held GPS indicates a ground speed of $100\,\mathrm{km/h}$. (Ground speed measures the horizontal speed of the glider relative to a stationary object on the ground.) At the same time, Rudolf reports running at $90\,\mathrm{km/h}$. What rate should the VSI show in m/min? (Hint: let x be the ground distance between Santa and Rudolf and y Santa's altitude. Find l in terms of x and y and use that the cable's length doesn't change.)



6. Let f be a function which has n roots and is differentiable $n-1$ times. Use induction and Rolle's Theorem to show that $\frac{d^{n-1}f}{dx^{n-1}}$ has a root.

7. When the Met Lounge charges \$x for a glass of beer during the Thu night happy hour, it sells $\frac{1000}{x^2+1/80}$ glasses. One glass of beer costs them 10 cents. What price maximizes the profit the Met Lounge rakes in on Thu night? Make sure you really found a maximum by doing either the 1st or the 2nd derivative test.