Math 1101 Quiz Problems 1

Prepare by 11/9/05

1. Problem 17 on p. 85
2. Problem 18 on p. 85
3. Prove by induction that for any $n \in \mathbb{Z}^+$,
$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$
4. Prove by induction that for any $n \in \mathbb{Z}^+$,
$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
5. Prove by induction that for any $n \in \mathbb{Z}^+$,
$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$
6. Prove by induction that for any integer $n \geq 2$,
$$\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\cdots\left(1 - \frac{1}{n}\right) = \frac{1}{n}$$
7. Prove by induction that for any $n \in \mathbb{Z}^+$,
$$2 \cdot 4 \cdot 6 \cdots (2n) \leq 1 \cdot 3 \cdot 5 \cdots (2n+1)$$
8. Prove by induction that for any integer $n \geq 5$,
$$2^n > n^2$$
9. Prove by induction that for any $n \in \mathbb{Z}^+$,
$$\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)} \geq \frac{1}{2n}$$
10. Problem 19 on p. 85
11. Problem 20 on p. 85
12. Recall that the Fibonacci sequence is defined by
$$F_1 = F_2 = 1$$
$$F_{n+1} = F_{n-1} + F_n \qquad n \geq 2$$
Prove that the following explicit formula allows to compute F_n without finding $F_1, F_2, \ldots, F_{n-1}$ first
$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$
13. Let
$$f_0(x) = 1$$
$$f_{n+1}(x) = x f_n(x) + 1 \qquad n \in \mathbb{N}$$
Compute the first few such functions and guess an expression for $f_n(x)$. Use induction to prove that your formula is correct.