Math 143 worksheet on areas and volumes

Oct 15, 2014

1. Carefully read and understand Examples 1 and 2 in Section 8.1. You will now use the same technique to find the formula for the area of a circle of radius r.

   First consider the upper semicircle of a circle of radius r. Cut it up into n horizontal slices of width w_i each and height Δh. Express w_i in terms of h, where h is the distance of the slice from the center of the circle. Draw a diagram that shows this.

2. If Δh is small, each such slice looks similar to a long and thin rectangle. Approximate the area of the slice with the area of such a rectangle and express the sum of the areas of the rectangles using summation notation.

3. Let the number of slices $n \to \infty$. As you do this $\Delta h \to 0$ and the sum from the previous part becomes a Riemann sum that is equal to an integral. Evaluate this integral using some appropriate integration technique. Double the result to get the familiar formula for the area of a circle.

4. Notice that you can think of an ellipse as a circle that is stretched along one of its diameters. Find a good way to modify the above calculation to compute the area of an ellipse of horizontal and vertical radii a and b.

5. Now read and understand Example 3 in Section 8.1. Use the same technique to find the volume of a right circular cone of radius r and height h. Do you get the familiar formula?

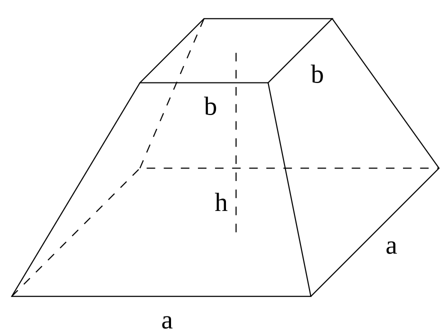
6. Would your computation in part 5 work to determine the volume of an oblique cone? If not, find a way to modify it so it does. If you don't know what an oblique cone is, look it up on Wikipedia.

7. The following computation of the volume of the frustum of a pyramid is from an ancient Egyptian text from about 1850 B.C called the Moscow papyrus.

   "You are given a truncated pyramid of 6 for the vertical height by 4 on the base by 2 on the top. You are to square this 4, result 16. You are to double 4, result 8. You are to square 2, result 4. You are to add the 16, the 8, and the 4, result 28. You are to take one third of 6, result 2. You are to take 28 twice, result 56. See, it is right. You will find it right."

   You will now check this computation by[finding the general formula for the volume of a truncated pyramid. First read and understand Example 5 in Section 8.1.

   Consider a truncated pyramid to the right, whose base is a square of side a, top is a square of side b, and the height is h. Use a computation similar to the one in Example 5 to find the volume of such a truncated pyramid.

8. Is the Egyptian formula in part 7 correct?

9. Read and understand Example 4 in Section 8.1. Do a similar computation to find the volume of a sphere of radius r. Do you get the familiar formula?

10. If you rotate a parabola about its axis of symmetry, you get what's called a circular paraboloid. If you have trouble visualizing this, Google "circular paraboloid." Consider a bowl whose shape is the circular paraboloid you get by rotating the parabola $y = (x/3)^2$ between $x = -6$ and $x = 6$ about its axis of symmetry. Suppose x and y are measured in inches. Find the volume of this bowl. (Hint: use horizontal slices.)