MATH 15 PRACTICE FINAL

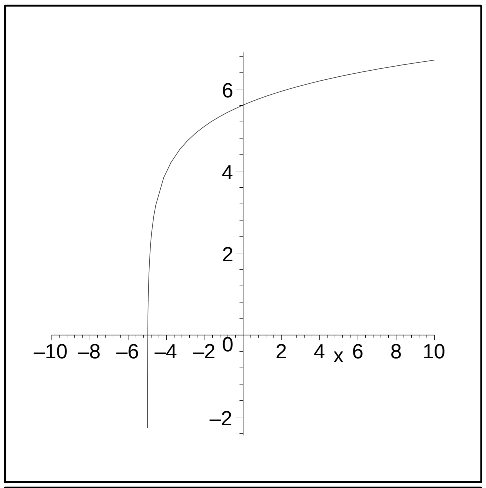
S'01, June 4, 2001

Unless told otherwise, your answers must be carefully justified. Remember that it is not enough to have a correct answer, you must show how you got it. Neat work, clear and to-the-point explanations will receive more credit than messy, chaotic answers. You may not use books, notes, and calculators on this exam, but you may have two 3" by 5" handwritten cheat sheets. You may leave logs, and numbers of the form $e^{\text{something}}$ in your answers, but not things like $\log 1$ or $e^{\ln 3}$.

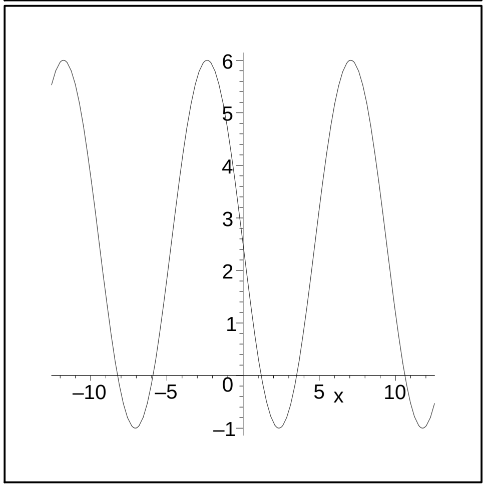
1. Find the domain and range of
$$f(x) = \log\left(\frac{x-1}{x+2}\right).$$
2. According to federal tax regulations, a student's parents can claim up to \$1500 in tax credit for the educational expenses if their annual income is less than \$80000. If it's more than \$100000, they are not eligible for the credit. Between \$80000 and \$100000, the credit is phased out linearly: for example, someone making \$90000 would be able to claim half of the credit, \$750. Find a piecewise defined function $f(x)$ that gives the amount of educational tax credit that some making \$x a year is allowed to claim. Graph $f(x)$. (You may want to do the graph first.)
3. Find the inverse of
$$f(x) = e^{\frac{x+2}{x-2}}$$
for $x > 2$. What are the domain and range of $f^{-1}(x)$?
4. A radioactive element A is decaying exponentially. After 5 days, its weight is 36 g, after another 10 days it is 16 g. Find a function $m(t)$ giving the mass of A as a function of time. How much of A was there initially? Find the half-life.
5. (a) Show
$$\log\frac{35}{36} = \log 7 + 3\log 5 - 2\log 3 - 2.$$
   (b) Rewrite
$$\ln\left(\frac{\sqrt[4]{x^3 y}}{y^2}\right)$$
   in terms of $a = \ln x$ and $b = \ln y$.
6. You deposit \$1000 in a bank at 12% annual interest compounded quarterly. How much money do you have at the end of the 3rd year? (You don't need to evaluate the number.) How would your answer change if the interest were compounded continuously? Which of these answers do you think is bigger?
7. The data in the following table are believed to satisfy either $y = ax^k$ or $y = ae^{kx}$. Plot two appropriate graphs to decide which model is better, then find a and k for that model.

x	y	$\ln x$	$\ln y$
8	23	2.08	3.14
17	150	2.83	5.01
23	496	3.14	6.21
26	860	3.26	6.76
32	2720	3.47	7.91
37	8051	3.61	8.99

8. On the right is a graph of the function $f(x)$.
   (a) Plot
   $$\frac{1}{2}f(3x-1)+2$$
   on a separate graph.
   (b) Is $f(x)$ invertible? Sketch a graph of $f^{-1}(x)$.



9.
   (a) Find the period, frequency, amplitude, and midline of $f(t) = 5\cos(3x + \pi/4) - 2$. Graph $f(t)$.
   (b) Find a sinusoidal function $f(x)$ that fits the graph on the right.



10. Fill in the following tables

x (radians)	0		$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$			$-\frac{5}{6}\pi$	$\frac{105\pi}{4}$
x (degrees)		$30°$				$120°$	$-135°$		
$\sin x$									
$\cos x$									
$\tan x$									

x	0	1	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\arcsin x$					
$\arccos x$					

11. (a) Find all possible angles θ such that $\cos\theta = 1/2$.
    (b) Find $\sin x$ and $\tan x$, if $\cos x = 1/3$.