Math 15 Practice Midterm

S'01, May 2, 2001

Unless told otherwise, your answers must be carefully justified. Remember that it is not enough to have a correct answer, you must show how you got it. Neat work, clear and to-the-point explanations will receive more credit than messy, chaotic answers. You may not use books, notes, and calculators on this exam, but you may have a 3" by 5" handwritten cheat sheet.

1. (a) You ride your bicycle from UCSB to Santa Barbara harbor (12 miles). Part of the way, you ride at 15 mph, then you walk your bike at 3 mph. Find a function $f(d)$ that gives the total travel time in hours as a function of d, the part of the distance walked (in miles).
   (b) Find $f(f(2))$ and $f^{-1}(3)$ if
$$f(x) = \frac{x+2}{x-1}.$$
2. (a) Find an equation of the line through $(2, 0)$ and $(-2, 2)$.
   (b) Find the intersection of $y = 3x + 1$ and $x + y = 1$. Check your answer.
3. (a) Show that
$$1 - 3\log 2 = \log\left(\frac{5}{4}\right)$$
   (b) Let $a = \log x$ and $b = \log y$. Express in terms of a and b
$$\log\left(\frac{\sqrt[3]{x^2}}{\sqrt[5]{y}}\right).$$
4. Graph the function
$$f(x) = \begin{cases} x+1 & \text{if } x < 0 \\ x-1 & \text{if } 0 \le x < 2 \\ x+1 & \text{if } 2 \le x \end{cases}.$$
   Find $f(f(3))$ and $f^{-1}(-2)$.
5. Find the domain and the range of
$$f(x) = \frac{2}{\sqrt{1-\frac{1}{x}}}.$$
6. You have some radioactive material whose mass is decaying exponentially. After two weeks, there is 200 g left, after five weeks only 25 g.
   (a) Find a function $f(t)$ that describes the amount of the material after t weeks.
   (b) How much material was there initially?
   (c) What percentage of the material decays in a week?
   (d) Assuming the decay were linear at a rate of 25%/week, how much of the material would you have after five weeks?