Math 2111 Quiz Problems 2
Prepare by 11/2/05

1. 3.4.27
2. 3.4.28
3. 3.5.35
4. Let V be a vector space and $U, W \subseteq V$ subspaces. Prove or disprove the following.
   (a) $U \cap W$ is a subspace of V.
   (b) $U \cup W$ is a subspace of V.
   (c) $U \setminus W$ is a subspace of V.
   (d) $U + W = \{u + w | u \in U, w \in W\}$ is a subspace of V.
5. Let V be a vector space and $S, T \subseteq V$. Prove or disprove the following.
   (a) $\text{span}(S) \cap \text{span}(T) = \text{span}(S \cap T)$
   (b) $\text{span}(S) \cup \text{span}(T) = \text{span}(S \cup T)$
   (c) If $S \subseteq T$, then $\text{span}(S) \subseteq \text{span}(T)$.
   (d) If $T \subseteq \text{span}(S)$, then $\text{span}(T) \subseteq \text{span}(S)$.
   (e) $\text{span}(\text{span}(S)) = \text{span}(S)$. (Hint: Use the previous statement.)
   (f) $\text{span}(S \cup T) = \text{span}(S) + \text{span}(T)$. (See the previous exercise on what it means to add two subspaces.)
6. Let V be a vector space and $S, T \subseteq V$. Prove or disprove the following.
   (a) If $S \subseteq T$ and T is linearly independent then S is linearly independent.
   (b) If $S \subseteq T$ and T is linearly dependent then S is linearly dependent.
   (c) If $S \subseteq T$ and S is linearly independent then T is linearly independent.
   (d) If $S \subseteq T$ and S is linearly dependent then T is linearly dependent.
   (e) If S and T are linearly independent then $S \cap T$ is linearly independent.
   (f) If S and T are linearly dependent then $S \cap T$ is linearly dependent.
   (g) If S and T are linearly independent then $S \cup T$ is linearly independent.
   (h) If S and T are linearly dependent then $S \cup T$ is linearly dependent.
7. Let V be a vector space and $U \subseteq W$ subspaces of V. Prove or disprove the following.
   (a) If S is a basis of U and T is a basis of W then $S \subseteq T$.
   (b) Given a basis S of U there always exists a basis T of V such that $S \subseteq T$.
   (c) Given a basis T of W there always exists a basis S of U such that $S \subseteq T$.
8. Let V be a vector space and U, W subspaces of V. Let S be a basis of U and T be a basis of W. Prove or disprove the following.
   (a) $S \cap T$ is a basis of some subspace of V.
   (b) $S \cap T$ is a basis of $U \cap W$.
   (c) $S \cup T$ is a basis of some subspace of V.
   (d) $S \cup T$ is a basis of $U + W$.