MATH 302 EXAM 1 SOLUTIONS
Feb 21, 2007

1. (10 pts) Consider the sentence "If the sky is blue and giraffes eat lions then elephants are pink or chickens have teeth."
   (a) Decide if the sentence is true or false. Carefully justify your answer.

   Since giraffes don't eat lions, the conjunction in the premise is false, which makes the conditional true.

   (b) Identify the parts of the above statement and translate it into symbols.

   $$P = \text{The sky is blue.}$$
   $$Q = \text{Giraffes eat lions.}$$
   $$R = \text{Elephants are pink.}$$
   $$S = \text{Chickens have teeth.}$$

   $$(P \wedge Q) \implies (R \vee S)$$

   Note that the lack of commas in the sentence indicates that you should do $P \wedge Q$ and $R \vee S$ before the conditional.

2. (13 pts)
   (a) Construct the complete truth table for $(P \vee Q) \implies (P \iff Q)$.

P	Q	$(P$	$\vee$	$Q)$	$\implies$	$(P$	$\iff$	$Q)$
T	T	T	T	T	T	T	T	T
T	F	T	T	F	F	T	F	F
F	T	F	T	T	F	F	F	T
F	F	F	F	F	T	F	T	F
		1	3	2	7	4	6	5

   (b) Use the truth table above to guess a simpler logical expression involving P and Q which is equivalent to $(P \vee Q) \implies (P \iff Q)$.

   Notice that the final column of the truth table is identical to that of $P \iff Q$. So $P \iff Q$ is equivalent to $(P \vee Q) \implies (P \iff Q)$.

   (c) Let $P = $ "Darth Vader is evil." and $Q = $ "Han Solo is a jedi.". Translate the logical expression $(P \vee Q) \implies (P \iff Q)$ into an English sentence. (Hint: Do you need commas? If you do, use them, if you don't, don't use them.)

   If Darth Vader is evil or Han Solo is a jedi then Darth Vader is evil if and only if Han Solo is a jedi.
   Note that there is no need for commas because the structure of the sentence makes is clear that $P \vee Q$ and $P \iff Q$ are to be evaluated before the conditional.

3. (15 pts)
   (a) Define what contradiction means.

   A contradiction is a statement that is always false by logical necessity.

(b) Decide if $(\neg P \implies Q) \iff \neg(P \vee Q)$ is a tautology, contradiction, or neither.

Here is the truth table:

P	Q	$(\neg P$	$\implies$	$Q)$	$\iff$	$\neg$	$(P$	$\vee$	$Q)$
T	T	F	T	T	F	F	T	T	T
T	F	F	T	F	F	F	T	T	F
F	T	T	T	T	F	F	F	T	T
F	F	T	F	F	F	T	F	F	F
		1	3	2	8	7	4	6	5

As column 8 shows, this is a contradiction.

(c) Prove that $(P \iff Q) \iff ((P \wedge Q) \vee \neg(P \vee Q))$ is an equivalence.

You could do this using a truth table. Let me do if differently for a change. Notice that $P \iff Q$ is true whenever P and Q have the same truth value and is false when they have different truth values. $P \wedge Q$ is true when P and Q are both true and is false otherwise, while $\neg(P \vee Q)$) is true exactly when P and Q are both false and is false otherwise. So the disjunction on the RHS will be true exactly when P and Q are both true or both false, and is false otherwise. This matches the truth value of $P \iff Q$, making the biconditional an equivalence.

4. (12 pts) Consider the statement "You won't stay dry if you jump into water." In the exercises below, you may convert double negatives to a less awkward form.
   (a) State the converse of the above statement.

   The key to these three problems is to identify the premise and the conclusion correctly. The premise in the above sentence is $P =$ "you jump into water" and the conclusion is $Q =$ "you won't stay dry". Hence the converse is

   $$Q \implies P = \text{If you won't stay dry then you jump into water.}$$

   (b) State the inverse of the above statement.

   $$\neg P \implies \neg Q = \text{If you don't jump into water then you'll stay dry.}$$

   (c) State the contrapositive of the above statement.

   $$\neg Q \implies \neg P = \text{If you'll stay dry then you don't jump into water.}$$

5. (10 pts) **Extra credit problem.** Recall that in the country of Logica, everyone is either a square shooter (always tells the truth) or a liar (always lies). Suppose that you are an inhabitant of Logica and are actually a liar. A crime is committed and you come under suspicion. It is known that the perpetrator was a liar. You come under suspicion for committing this crime and are tried in court.
   (a) Suppose you are actually innocent of the crime. According to the local customs, you are allowed to make only one statement in your defense. What do you say?

   You could say "I committed this crime." Firstly, this is a lie, so as a liar, you can say this. Secondly, it actually proves your innocence. The jury will think if you were indeed

the perpetrator, you'd have to be a liar, but then you wouldn't admit that you are guilty. So you can't actually be the perpetrator and they'll acquit you.

(b) Suppose you are in fact guilty of the crime. Again, local customs allow you to make only one statement in your defense. Is there anything you can say that would convince the jury that you are innocent? Assume that the jury are smart and think rationally.

There is nothing you can say that would convince them. To convince them that you are innocent, you'd have to say something that the perpetrator of the crime couldn't possibly say. But you are the perpetrator of the crime, so you can't say anything the perpetrator can't say.

The lesson to learn from this is that you can't prove true something that is actually false using logic alone.