MATH 302 EXAM 1 Solutions Feb 18, 2009

- 1. (5 pts each) Is the statement a tautology, a contradiction, or neither?
 - (a) If Ethel goes to the movies then Agnes will eat a cake, and Agnes does not eat a cake, and Ethel goes to the movies.

Let P = "Ethel goes to the movies" and Q = "Agnes eats cake." Then this statement is $((P \to Q) \land \neg Q) \land P)$. The truth table is

((P	\rightarrow	Q)	\wedge	$\neg Q)$	\wedge	P
T	Т	T	F	F	F	T
T	F	F	F	T	F	T
F	T	T	F	F	F	F
F	T	F	T	T	F	F
1	3	2	5	4	7	6

Hence this statement is a contradiction.

(b) Rabbits eat cake or pie, and if rabbits eat pie then they eat cake.

Let P = "Rabbits eat cake" and Q = "Rabbits eat pie." Then this statement is $(P \lor Q) \land (Q \to P)$. The truth table is

(P	\vee	Q)	\wedge	(Q	\rightarrow	P)
T	T	T	T	T	T	T
T	T	F	T	F	T	T
F	T	T	F	T	F	F
F	F	F	F	F	T	F
1	3	2	7	4	6	5

Hence this statement is neither a tautology nor a contradiction.

2. (10 pts each) For each of the following arguments, if it is valid, give a derivation, and if it is not valid, show why.

(a)

$$\begin{array}{c} \neg X \to Y \\ \neg X \to Z \\ \hline \hline \neg Z \to \neg Y \end{array}$$

This is an invalid argument. If X and Y are true and Z is false, the premises are true and the conclusion is false.

$(1) L \to M$	
$(2) (M \lor N) \to (L \to K)$	
(3) $\neg P \land L$	
(4) L	(3), Simplification
(5) M	(1), (4), Modus Ponens
(6) $M \lor N$	(5), Addition
(7) $L \to K$	(2), (6), Modus Ponens
(8) K	(7), (4), Modus Ponens
	$(1) L \to M$ $(2) (M \lor N) \to (L \to K)$ $(3) \neg P \land L$ $(4) L$ $(5) M$ $(6) M \lor N$ $(7) L \to K$ $(8) K$



- 3. (10 pts each)
 - (a) Find the converse, inverse, and contrapositive of the following statement. Hot dogs are the best if they are wrapped in a slice of bacon and served with pineapples and maraschino cherries.

Let

P = "Hot dogs are the best."

Q = "Hot dogs are wrapped in a slice of bacon and

served with pineapples and maraschino cherries."

Then the condition above is $Q \to P$. In the inverse and the contrapositive, we will need $\neg Q$, which can be stated as "It is not the case that hot dogs are wrapped in a slice of bacon and served with pineapples and maraschino cherries" or as its logical equivalent "Hot dogs are not wrapped in a slice of bacon or not served with pineapples and maraschino cherries." I will use the latter since it sounds more natural. You could even state $\neg Q$ as "Hot dogs are not wrapped in a slice of bacon or not served with pineapples or not served with maraschino cherries."

Converse $(P \rightarrow Q)$: If hot dogs are the best, then they are wrapped in a slice of bacon and served with pineapples and maraschino cherries.

Inverse $(\neg Q \rightarrow \neg P)$: Hot dogs are not the best if they are not wrapped in a slice of bacon or not served with pineapples and maraschino cherries.

Contrapositive $(\neg P \rightarrow \neg Q)$: If hot dogs are not the best, then they are not wrapped in a slice of bacon or not served with pineapples and maraschino cherries.

(b) Prove that a conditional statement is equivalent to its contrapositive, but not to its converse.

You can compare the truth tables of $P \to Q$, $\neg Q \to \neg P$, and $Q \to P$:

P	Q	$P \to Q$	P	Q	$\neg Q$	\rightarrow	$\neg P$	P	Q	$Q \to P$
T	T	Т	T	T	F	T	F	T	T	Т
T	F	F	T	F	T	F	F	T	F	T
F	T	T	F	T	F	T	T	F	T	F
F	F	T	F	F	T	T	T	F	F	T

The first two are the same, but the third one is different.

4. (10 pts) Construct a verbal argument which is invalid. Prove that your argument is indeed invalid.

Here is a very simple example. Homer Simpson is fat or dumb. He is fat. Therefore he is not dumb.

Let P = "Homer Simpson is fat" and Q = "Homer Simpson is dumb". Then the argument is

$$P \lor Q$$

 P

If P is true and Q is false, the premises are true and the conclusion is false. Therefore this argument is invalid.

If you want a more sophisticated example, look at pp. 38-39 in your textbook or try Exercise 1.4.2.(5).

5. (10 pts) Extra credit problem.

Recall that in the country of Logica, there are two kinds of people: square shooters, who always tell the truth, and liars, who always lie. In a remote part of that country, lives a group of people who speak an unusual local dialect: instead of yes and no, they say boo and bah. Unfortunately, we don't know which of these means yes and which means no. Other than this little quirk, they understand and speak English just fine. You meet one of these locals and he seems like a nice guy, but he could be either a square shooter or a liar. Surely, you'd like to know. How can you tell by asking him only one boo/bah question?

For starters, suppose that the local you ran into spoke regular English. Even then you wouldn't want to ask him "Are you a square shooter?" because he'd have to answer yes whether he is a square shooter or a liar. (You could instead ask him "Is 1 + 0 = 1?" or if you don't want to assume that he knows basic math, "Do you exist?") But suppose you asked him "Would you say you are a square shooter?" To you, this may sound like the same question. But to a native of Logica (a Logican?), it doesn't. If he is a square shooter, he would indeed say he is a square shooter (since he is lying), but he cannot admit that he would say so, therefore he will answer no.

Now if the local in question only speaks the boo/bah dialect, you can model your question on the last one. Ask him "Would you say boo if I asked you whether you are a square shooter?" First, suppose boo means yes. Then a square shooter would in fact say boo if asked whether he was a square shooter, and he will honestly tell you that this is what he'd say, so he will answer boo. A liar would still answer boo if asked whether he was a square shooter (because he is lying), but he will want to deny this, so he will answer bah to your tricky question.

Now, suppose boo means no. Then a square shooter wouldn't want to say boo if asked if he was a square shooter. So he will want to give a negative answer to your question, that is, he will say boo. A liar wouldn't answer boo if asked whether he was a square shooter either, but he will lie about this, and will claim that he would answer boo. That is, he will give a positive answer, which in his dialect is bah.

Whichever way boo and bah might go, an answer of boo to your question tells you that you are talking to a square shooter and an answer of bah means he is a liar.

There are many other questions that would solve this problem. Can you come up with another?

BTW, it is tempting to ask the local the simple question "Does boo mean yes?" This would work fine too if you are certain that he understands the word yes. But if he only speaks his local dialect of boo/bah, then the word yes may be meaningless to him.