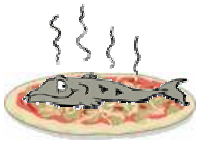
Math 302 Exam 2 Solutions

Mar 25, 2009

1. (10 pts) Write a derivation of the following argument.

(1)	$(\forall\ a \text{ in } V)[N(a) \to B(a)]$	
(2)	$(\exists\ b \text{ in } V)[N(b) \wedge D(b)]$	
(3)	$N(x) \wedge D(x)$	for some $x \in V$, (2), Existential instantiation
(4)	$N(x)$	(3), Simplification
(5)	$N(x) \to B(x)$	(1), Universal instantiation
(6)	$B(x)$	(5), (4), Modus Ponens
(7)	$D(x)$	(3), Simplification
(8)	$B(x) \wedge D(x)$	(6), (7), Adjunction
(9)	$(\exists\ c \text{ in } V)[B(c) \wedge D(c)]$	(8), Existential generalization

2. (10 pts) Write a derivation for the following valid argument. State (and justify) if the premises are consistent or inconsistent.



If Marcus likes pizza then he likes beer. If Marcus likes beer then he does not like herring. If Marcus likes pizza then he likes herring. Marcus likes pizza. Therefore he likes herring pizza.

Let

$P =$ "Marcus likes pizza."
$Q =$ "Marcus likes beer."
$R =$ "Marcus likes herring."
$S =$ "Marcus likes herring pizza."

Then the above argument is

(1) $P \to Q$
(2) $Q \to \neg R$
(3) $P \to R$
(4) P

It is easy to see that the premises are inconsistent. Since P is true, Q and R must be true to make $P \to Q$ and $P \to R$ true. But then $Q \to \neg R$ cannot possibly be true. Here is a derivation:

(5)	Q	(1), (4), Modus Ponens
(6)	$\neg R$	(2), (5), Modus Ponens
(7)	R	(3), (4), Modus Ponens
(8)	$R \vee S$	(7), Addition
(9)	S	(8), (7), Modus Tollendo Ponens

3. (10 pts) Negate the following statement: For every real number $\epsilon > 0$, there exists a positive integer k such that for all positive integers n, it is the case that $|a_n - k^2| < \epsilon$.

Use the usual properties of negating quantified statements. Don't just add "it is not the case" or "it is not true" to the beginning of the sentence. If you do this right, you shouldn't even need to use the word no/not. (Hint: it may help if you first convert the statement to symbols, then negate, then convert back to words.)

Here is the statement in symbols:

$$(\forall \epsilon \in \mathbb{R}^+)(\exists k \in \mathbb{Z}^+)(\forall n \in \mathbb{Z}^+)|a_n - k^2| < \epsilon$$

Here is the negation:

$$\begin{aligned}
&\neg[(\forall \epsilon \in \mathbb{R}^+)(\exists k \in \mathbb{Z}^+)(\forall n \in \mathbb{Z}^+)|a_n - k^2| < \epsilon] \\
&\iff (\exists \epsilon \in \mathbb{R}^+)(\forall k \in \mathbb{Z}^+)(\exists n \in \mathbb{Z}^+)\neg(|a_n - k^2| < \epsilon) \\
&\iff (\exists \epsilon \in \mathbb{R}^+)(\forall k \in \mathbb{Z}^+)(\exists n \in \mathbb{Z}^+)|a_n - k^2| \geq \epsilon
\end{aligned}$$

In words: there exists a real number $\epsilon > 0$ such that for every positive integer k there is a positive integer n such that $|a_n - k^2| \geq \epsilon$.

4. (10 pts) Make up a verbal argument which could be described by the symbols in problem 1. Show how your argument fits the symbols, i.e. be sure to describe what V, N, B, and D are.

Every animal that has feathers is a bird. There is an animal which has feathers and cannot fly. Therefore there is a bird which cannot fly.

$$\begin{aligned}
V &= \text{"Set of all animals."} \\
N(x) &= \text{"}x\text{ has feathers."} \\
B(x) &= \text{"}x\text{ is a bird."} \\
D(x) &= \text{"}x\text{ cannot fly."}
\end{aligned}$$

5. (5 pts each) For this exercise, you may want to refer to the Block World syntax help page that is attached to the end of this exam.
   (a) Translate the following sentence into a statement that Block World would understand. Every medium triangle is smaller than every other object.

```
A x A y ((Medium(x) /\ Triangle(x) /\ x<>y) => Smaller(x,y))
```

   (b) Translate the following sentence into a statement that Block World would understand. There are no triangle and pentagon that are of the same size.

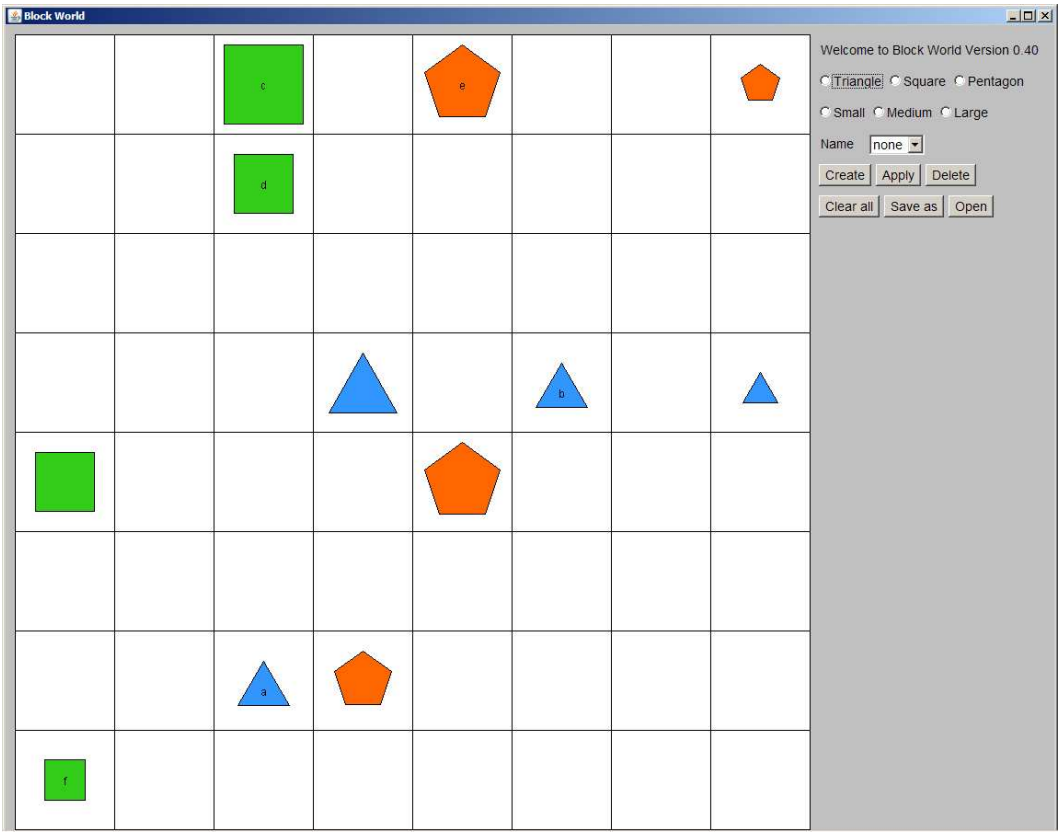
```
A x A y ((Triangle(x) /\ Pentagon(y)) => ~SameSize(x,y))
```

   (c) What does the following statement say <u>in plain English</u>?

```
A x ((Square(x) /\ Small(x)) <=> (x=f))
```

It says that x is a small square if and only if x is actually f. In plain English, f is the only small square on the board.

   (d) What <u>two</u> changes would you have to do to the board below to make the statement in part (a) true? Be sure to jutify your answer.



You need to remove the two medium triangles from the board, or change them into something other than medium triangles. Then there will be no medium triangle that is smaller than the large objects on the board, so the statement will become true.

6. (10 pts) **Extra credit problem.** Translate the following sentence into a statement that Block World would understand. Explain how and why your statement should work.

   Between any two distinct squares, there is a unique triangle.

   One way to say this is

```
A x A y ((Square(x) /\ Square(y) /\ x<>y) =>
(E z (Triangle(z) /\ Between(z,y,x) /\
~(E u (Triangle(u) /\ Between(u,x,y) /\ u<>z)))))
```

   Here is how it works (I left out most parentheses to make this more legible):

`A x A y`	For every object x and every object y
`(Square(x) /\ Square(y) /\ x<>y)`	if x is a square and y is a square and they are not the same
`=>`	then
`E z`	there is an object z such that
`Triangle(z) /\`	z is a triangle and
`Between(z,x,y) /\`	z is between x and y and
`~`	it is not true that
`E u`	there is an object u such that
`Triangle(u) /\`	u is a triangle and
`Between(u,x,y) /\`	u is between x and y and
`u<>z`	u is different from the z we've already found.