1.1 EXERCISES

In Exercises 1–12, determine whether the reasoning is an example of deductive or inductive reasoning.

1. If the mechanic says that it will take two days to repair your car, then it will actually take four days. The mechanic says, “I figure it’ll take a couple of days to fix it, ma’am.” Then you can expect it to be ready four days from now.

2. If you take your medicine, you’ll feel a lot better. You take your medicine. Therefore, you’ll feel a lot better.

3. It has rained every day for the past five days, and it is raining today as well. So it will also rain tomorrow.

4. Natalie’s first three children were boys. If she has another baby, it will be a boy.

5. Josh had 95 Pokémon trading cards. Margaret gave him 20 more for his birthday. Therefore, he now has 115 of them.

6. If the same number is subtracted from both sides of a true equation, the new equation is also true. I know that $9 + 18 = 27$. Therefore, $(9 + 18) - 12 = 27 - 12$.

7. If you build it, they will come. You build it. So, they will come.

8. All men are mortal. Socrates is a man. Therefore, Socrates is mortal.

9. It is a fact that every student who ever attended Brainchild University was accepted into medical school. Since I am attending Brainchild, I can expect to be accepted to medical school, too.

10. For the past 25 years, a rare plant has bloomed in Columbia each summer, alternating between yellow and green flowers. Last summer, it bloomed with green flowers, so this summer it will bloom with yellow flowers.

11. In the sequence $5, 10, 15, 20, \ldots$, the most probable next number is 25.

12. Britney Spears’ last four single releases have reached the Top Ten list, so her current release will also reach the Top Ten.

13. Discuss the differences between inductive and deductive reasoning. Give an example of each.

14. Give an example of faulty inductive reasoning.

Determine the most probable next term in each list of numbers.

EXERCISES

15. 6, 9, 12, 15, 18

16. 13, 18, 23, 28, 33

17. 3, 12, 48, 192, 768

18. 32, 16, 8, 4, 2

19. 3, 6, 9, 15, 24, 39

20. 1/3, 3/5, 5/7, 7/9, 9/11

21. 1/2, 3/4, 5/6, 7/8, 9/10

22. 1, 4, 9, 16, 25

23. 1, 8, 27, 64, 125

24. 2, 6, 12, 20, 30, 42

25. 4, 7, 12, 19, 28, 39

26. $-1, 2, -3, 4, -5, 6$

27. 5, 3, 5, 5, 3, 5, 5, 5, 3, 5, 5, 5, 5, 3, 5, 5, 5, 5

28. 8, 2, 8, 2, 2, 8, 2, 2, 2, 8, 2, 2, 2, 2, 8, 2, 2, 2, 2

29. Construct a list of numbers similar to those in Exercise 15 such that the most probable next number in the list is 60.

30. Construct a list of numbers similar to those in Exercise 26 such that the most probable next number in the list is 9.

In Exercises 31–42, a list of equations is given. Use the list and inductive reasoning to predict the next equation, and then verify your conjecture.

31.
$$\begin{aligned}(9 \times 9) + 7 &= 88\\ (98 \times 9) + 6 &= 888\\ (987 \times 9) + 5 &= 8888\\ (9876 \times 9) + 4 &= 88{,}888\end{aligned}$$

32.
$$\begin{aligned}(1 \times 9) + 2 &= 11\\ (12 \times 9) + 3 &= 111\\ (123 \times 9) + 4 &= 1111\\ (1234 \times 9) + 5 &= 11{,}111\end{aligned}$$

33.
$$\begin{aligned}3367 \times 3 &= 10{,}101\\ 3367 \times 6 &= 20{,}202\\ 3367 \times 9 &= 30{,}303\\ 3367 \times 12 &= 40{,}404\end{aligned}$$

34.
$$\begin{aligned}15873 \times 7 &= 111{,}111\\ 15873 \times 14 &= 222{,}222\\ 15873 \times 21 &= 333{,}333\\ 15873 \times 28 &= 444{,}444\end{aligned}$$

35.
$$\begin{aligned}34 \times 34 &= 1156\\ 334 \times 334 &= 111{,}556\\ 3334 \times 3334 &= 11{,}115{,}556\end{aligned}$$

36.
$$\begin{aligned}11 \times 11 &= 121\\ 111 \times 111 &= 12{,}321\\ 1111 \times 1111 &= 1{,}234{,}321\end{aligned}$$

37.
$$\begin{aligned}3 &= \frac{3(2)}{2}\\ 3 + 6 &= \frac{6(3)}{2}\\ 3 + 6 + 9 &= \frac{9(4)}{2}\\ 3 + 6 + 9 + 12 &= \frac{12(5)}{2}\end{aligned}$$

38.
$$\begin{aligned}2 &= 4 - 2\\ 2 + 4 &= 8 - 2\\ 2 + 4 + 8 &= 16 - 2\\ 2 + 4 + 8 + 16 &= 32 - 2\end{aligned}$$

39.
$$\begin{aligned}5(6) &= 6(6 - 1)\\ 5(6) + 5(36) &= 6(36 - 1)\\ 5(6) + 5(36) + 5(216) &= 6(216 - 1)\\ 5(6) + 5(36) + 5(216) + 5(1296) &= 6(1296 - 1)\end{aligned}$$

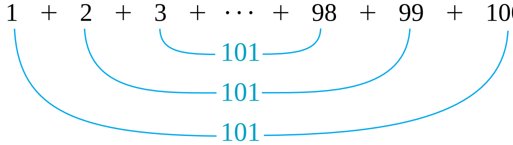
40.
$$\begin{aligned}3 &= \frac{3(3 - 1)}{2}\\ 3 + 9 &= \frac{3(9 - 1)}{2}\\ 3 + 9 + 27 &= \frac{3(27 - 1)}{2}\\ 3 + 9 + 27 + 81 &= \frac{3(81 - 1)}{2}\end{aligned}$$

41.
$$\begin{aligned}\frac{1}{2} &= 1 - \frac{1}{2}\\ \frac{1}{2} + \frac{1}{4} &= 1 - \frac{1}{4}\\ \frac{1}{2} + \frac{1}{4} + \frac{1}{8} &= 1 - \frac{1}{8}\\ \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} &= 1 - \frac{1}{16}\end{aligned}$$

42.
$$\begin{aligned}\frac{1}{1 \cdot 2} &= \frac{1}{2}\\ \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} &= \frac{2}{3}\\ \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} &= \frac{3}{4}\\ \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} &= \frac{4}{5}\end{aligned}$$

A story is often told about how the great mathematician Carl Friedrich Gauss (1777–1855) at a very young age was told by his teacher to find the sum of the first 100 *counting numbers. While his classmates toiled at the problem, Carl simply wrote down a single number and handed it in to his teacher. His answer was correct. When asked how he did it, the young Carl explained that he observed that there were* 50 *pairs of numbers that each added up to* 101*. (See below.) So the sum of all the numbers must be* $50 \times 101 = 5050$.

$$1 + 2 + 3 + \cdots + 98 + 99 + 100$$



$$50 \text{ sums of } 101 = 50 \times 101 = 5050$$

Use the method of Gauss to find each of the following sums.

43. $1 + 2 + 3 + \cdots + 200$

44. $1 + 2 + 3 + \cdots + 400$

45. $1 + 2 + 3 + \cdots + 800$

46. $1 + 2 + 3 + \cdots + 2000$

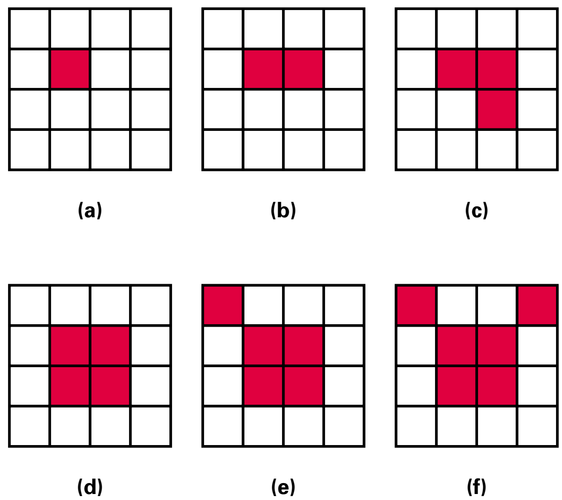
47. Modify the procedure of Gauss to find the sum $1 + 2 + 3 + \cdots + 175$.

48. Explain in your own words how the procedure of Gauss can be modified to find the sum $1 + 2 + 3 + \cdots + n$, where n is an odd natural number. (An odd natural number, when divided by 2, leaves a remainder of 1.)

49. Modify the procedure of Gauss to find the sum $2 + 4 + 6 + \cdots + 100$.

50. Use the result of Exercise 49 to find the sum $4 + 8 + 12 + \cdots + 200$.

51. Find a pattern in the following figures and use inductive reasoning to predict the next figure.



52. Consider the following table.

0	2	2	2	0	0	0	0	0
0	2	4	6	4	2	0	0	0
0	2	6	12	14	12	6	2	0
0	2	8	20	32	38	32	20	8

Find a pattern and predict the next row of the table.

53. What is the most probable next number in this list? 12, 1, 1, 1, 2, 1, 3 (*Hint:* Think about a clock.)

54. What is the next term in this list? O, T, T, F, F, S, S, E, N, T (*Hint:* Think about words and their relationship to numbers.)

55. **(a)** Choose any three-digit number with all different digits. Now reverse the digits, and subtract the smaller from the larger. Record your result. Choose another three-digit number and repeat this process. Do this as many times as it takes for you to see a pattern in the different results you obtain. (*Hint:* What is the middle digit? What is the sum of the first and third digits?)

(b) Write an explanation of this pattern. You may wish to use this exercise as a "number trick" to amuse your friends.

56. Choose any number, and follow these steps.

(a) Multiply by 2.
(b) Add 6.
(c) Divide by 2.
(d) Subtract the number you started with.
(e) Record your result.

Repeat the process, except in Step (b), add 8. Record your final result. Repeat the process once more, except in Step (b), add 10. Record your final result.

(f) Observe what you have done; use inductive reasoning to explain how to predict the final result. You may wish to use this exercise as a "number trick" to amuse your friends.

57. Complete the following.

$142{,}857 \times 1 =$ _____
$142{,}857 \times 2 =$ _____
$142{,}857 \times 3 =$ _____
$142{,}857 \times 4 =$ _____
$142{,}857 \times 5 =$ _____
$142{,}857 \times 6 =$ _____

What pattern exists in the successive answers? Now multiply 142,857 by 7 to obtain an interesting result.

58. Complete the following.

$12{,}345{,}679 \times 9 =$ _____
$12{,}345{,}679 \times 18 =$ _____
$12{,}345{,}679 \times 27 =$ _____

By what number would you have to multiply 12,345,679 in order to get an answer of 888,888,888?

59. Refer to Figures 2(b)–(e) and 3. Instead of counting interior regions of the circle, count the chords formed. Use inductive reasoning to predict the number of chords that would be formed if seven points were used.

60. The following number trick can be performed on one of your friends. It was provided by Dr. George DeRise of Thomas Nelson Community College.

(a) Ask your friend to write down his or her age. (Only whole numbers are allowed.)
(b) Multiply the number by 4.
(c) Add 10.
(d) Multiply by 25.
(e) Subtract the number of days in a non-leap year.
(f) Add the amount of change (less than a dollar, in cents) in his or her pocket.
(g) Ask your friend for the final answer.

If you add 115 to the answer, the first two digits are the friend's age, and the last two give the amount of change.

61. Explain how a toddler might use inductive reasoning to decide on something that will be of benefit to him or her.

62. Discuss one example of inductive reasoning that you have used recently in your life. Test your premises and your conjecture. Did your conclusion ultimately prove to be true or false?