1.4 Calculating, Estimating, and Reading Graphs

The photograph shows the Texas Instruments TI-1795 SV, a typical four-function calculator.

Since the introduction of hand-held calculators in the early 1970s, the methods of everyday arithmetic have been drastically altered. One of the first consumer models available was the Texas Instruments SR-10, which sold for nearly $150 in 1973. It could perform the four operations of arithmetic and take square roots, but could do very little more.

The search for easier ways to calculate and compute has culminated in the development of hand-held calculators and computers. This text assumes that all students have access to calculators, allowing them to spend more time on the conceptual nature of mathematics and less time on computation with paper and pencil. For the general population, a calculator that performs the operations of arithmetic and a few other functions is sufficient. These are known as **four-function calculators.** Students who take higher mathematics courses (engineers, for example) usually need the added power of **scientific calculators.** **Graphing calculators,** which actually plot graphs on small screens, are also available. Since calculators differ from one manufacturer to another, remember the following.

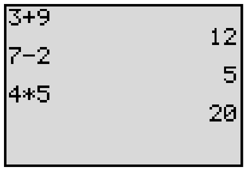
> Always refer to your owner's manual if you need assistance in performing an operation with your calculator. If you need further help, ask your instructor or another student who is using the same model.

Calculating Graphing calculators have become the standard in the world of advanced hand-held calculators. One of the main advantages of a graphing calculator is that both the information the user inputs into the calculator and the result

Any calculator (particularly a graphing calculator) consists of two components: the electronic “box” and the owner’s manual that explains how to use it.

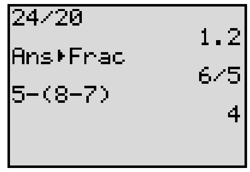
generated by that calculator can be viewed on the same screen. In this way, the user can verify that the information entered into the calculator is correct. While it is not necessary to have a graphing calculator to study the material presented in this text, we occasionally include graphing calculator screens to support results obtained or to provide supplemental information.*

The screens that follow illustrate some common entries and operations.



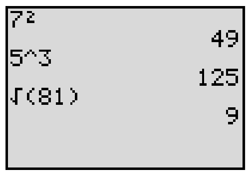
$3 + 9 = 12$
$7 - 2 = 5$
$4 \times 5 = 20$

A



$\frac{24}{20} = 1.2$
$1.2 = \frac{6}{5}$
$5 - (8 - 7) = 4$

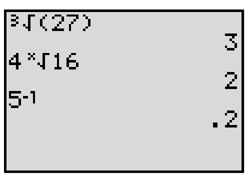
B



$7^2 = 49$
$5^3 = 125$
$\sqrt{81} = 9$

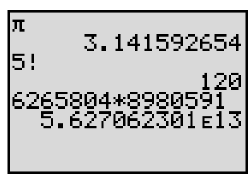
C

Screen A illustrates how two numbers can be added, subtracted, or multiplied. Screen B shows how two numbers can be divided, how the decimal quotient (stored in the memory cell Ans) can be converted into a fraction, and how parentheses can be used in a computation. Screen C shows how a number can be squared, how it can be cubed, and how its square root can be taken.



$\sqrt[3]{27} = 3$
$\sqrt[4]{16} = 2$
5^{-1} (or $\frac{1}{5}$) $= .2$

D



≈ indicates “is approximately equal to”

$\pi \approx 3.141592654$
5! (or $1 \times 2 \times 3 \times 4 \times 5$) $= 120$
$6{,}265{,}804 \times 8{,}980{,}591 \approx 5.627062301 \times 10^{13}$

E

Screen D shows how higher roots (cube root and fourth root) can be found, and how the reciprocal of a number can be found using -1 as an exponent. Screen E shows how π can be accessed with its own special key, how a *factorial* (as indicated by !) can be found and how a result might be displayed in *scientific notation.* (The

*Because it is one of the most popular models available, we include screens generated by TI-Graph Link™ software for the Texas Instruments TI-83 Plus calculator.

"E13" following 5.627062301 means that this number is multiplied by 10^{13}. This answer is still only an approximation, because the product $6{,}265{,}804 \times 8{,}980{,}591$ contains more digits than the calculator can display.)

Estimation While calculators can make life easier when it comes to computations, many times we need only estimate an answer to a problem, and in these cases a calculator may not be necessary or appropriate.

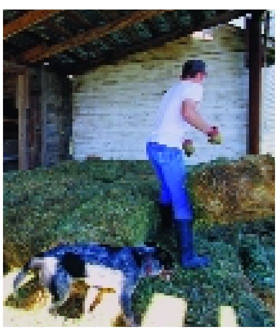
EXAMPLE 1 A birdhouse for swallows can accommodate up to 8 nests. How many birdhouses would be necessary to accommodate 58 nests?

If we divide 58 by 8 either by hand or with a calculator, we get 7.25. Can this possibly be the desired number? Of course not, since we cannot consider fractions of birdhouses. Do we need 7 or 8 birdhouses? To provide nesting space for the nests left over after the 7 birdhouses (as indicated by the decimal fraction), we should plan to use 8 birdhouses. In this problem, we must round our answer *up* to the next counting number. ■

EXAMPLE 2 In 2001, David Boston of the Arizona Cardinals caught 98 passes for a total of 1598 yards. What was his approximate average number of yards per catch?

Since we are asked only to find David's approximate average, we can say that he caught about 100 passes for about 1600 yards, and his average was approximately $1600/100 = 16$ yards per catch. (A calculator shows that his average to the nearest tenth was 16.3 yards. Verify this.) ■

EXAMPLE 3 In a recent year there were approximately 127,000 males in the 25–29-year age bracket working on farms. This represented part of the total of 238,000 farm workers in that age bracket. Of the 331,000 farm workers in the 40–44-year age bracket, 160,000 were males. Without using a calculator, determine which age bracket had a larger proportion of males.



Here, it is best to think in terms of thousands instead of dealing with all the zeros. First, let us analyze the age bracket 25–29 years. Since there were a total of 238 thousand workers, of which 127 thousand were males, there were $238 - 127 = 111$ thousand female workers. Here, more than half of the workers were males. In the 40–44-year age bracket, of the 331 thousand workers, there were 160 thousand males, giving $331 - 160 = 171$ thousand females, meaning fewer than half were males. A comparison, then, shows that the 25–29-year age bracket had the larger proportion of males. ■

Reading Graphs Using graphs has become an efficient means of transmitting information in a concise way. Any issue of the newspaper *USA Today* will verify this. There are many ways to represent information using graphs and charts. Pie charts, bar graphs, and line graphs are the most common.

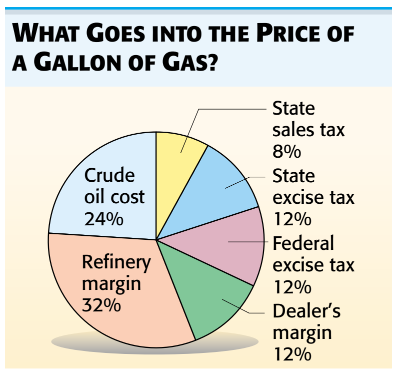
YOU TRY IT

EXAMPLE 4 Use the graphs in Figures 9–11 to make interpretations.

The pie chart in Figure 9 shows how the price of a gallon of gasoline in California is divided among various manufacturers and agencies. The sectors (resembling slices of pie) are sized to match how the price is divided. For example, most of the price (32%) goes to the refinery, while the least portion (8%) goes for state income tax. As expected, the percents total 100%. If the price of a gallon of gasoline is \$1.50, for example, the dealer's margin would be 12%, or $\$1.50 \times .12 = \$.18$.

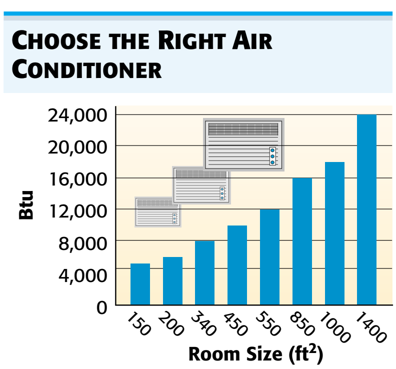
The bar graph in Figure 10 shows the relationship between the size of a room in square feet and the suggested size of air conditioner, in British thermal units (Btu). To illustrate, if we want to determine the size of air conditioner needed for an 850-square-foot room, read across at the bottom to find 850. The bar corresponding to 850 feet has a height corresponding to 16,000, found on the vertical scale at the left. We should use a 16,000 Btu air conditioner.

The line graph in Figure 11 shows the profits of CNBC from 1994 to 1999. It is read in a manner similar to the bar graph. In 1996, for example, profits were 80 million dollars, while in 1997, they were 130 million dollars. The fact that the line segments always rise from left to right indicates that profits increased each year in that time period.



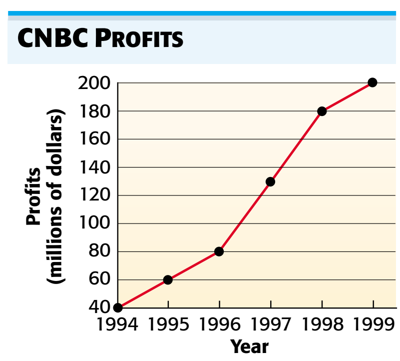
Source: California Energy Commission, 1999.

FIGURE 9



Source: Carey, Morris and James, Home Improvement for Dummies, IDG Books.

FIGURE 10



Source: Fortune, May 24, 1999, p. 142.

FIGURE 11

FOR FURTHER THOUGHT

Are You "Numerate"?

Letter is to *number* as *literacy* is to *numeracy.* In recent years much has been written about how important it is that the general population be "numerate." The essay "Quantity" by James T. Fey in *On the Shoulders of Giants: New Approaches to Numeracy* contains the following description of an approach to numeracy.

> Given the fundamental role of quantitative reasoning in applications of mathematics as well as the innate human attraction to numbers, it is not surprising that number concepts and skills form the core of school mathematics. In the earliest grades all children start on a mathematical path designed to develop computational procedures of arithmetic together with corresponding conceptual understanding that is required to solve quantitative problems and make informed decisions. Children learn many ways to describe quantitative data and relationships using numerical, graphic, and symbolic representations; to plan arithmetic and algebraic operations and to execute those plans using effective procedures; and to interpret quantitative information, to draw inferences, and to test the conclusions for reasonableness.

For Group Discussion

With calculator in hand, each student in the class should attempt to fill in the boxes with the digits 3, 4, 5, 6, 7, or 8, with each digit used at most once. Then take a poll to see who was able to come up with the closest number to the "goal number." You are allowed one minute per round.
Good luck!

Round I □ × □ □ □ □ = 30,000

Round II □ × □ □ □ □ = 40,000

Round III □ × □ □ □ □ = 50,000

Round IV □ □ × □ □ □ □ = 30,000

Round V □ □ × □ □ □ □ = 60,000