COLLABORATIVE INVESTIGATION

Discovering Mathematics in Pascal's Triangle

One fascinating array of numbers, *Pascal's Triangle*, consists of rows of numbers, each of which contains one more entry than the one before. The first five rows are shown here.

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          1
        1   1
      1   2   1
    1   3   3   1
  1   4   6   4   1
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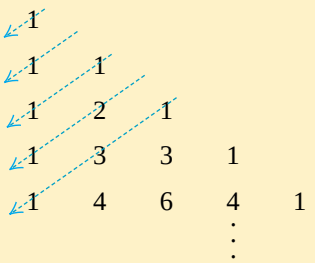
To discover some of its patterns, divide the class into groups of four students each. Within each group designate one student as A, one as B, one as C, and one as D. Then perform the following activities in order.

1. Discuss among group members some of the properties of the triangle that are obvious from observing the first five rows shown.
2. It is fairly obvious that each row begins and ends with 1. Discover a method whereby the other entries in a row can be determined from the entries in the row immediately above it. (*Hint:* In the fifth row, $6 = 3 + 3$.) Then, as a group, find the next four rows of the triangle, and have each member prepare his or her own copy of the entire first nine rows for later reference.
3. Now each student in the group will investigate a particular property of the triangle. In some cases, a calculator will be helpful. All students should begin working at the same time. (A discussion will follow.)
   Student A: Find the sum of the entries in each row. Notice the pattern that emerges. Now write the tenth row of the triangle.
   Student B: Investigate the successive differences in the diagonals from upper left to lower right. For example, in the diagonal that begins 1, 2, 3, 4, …, the successive differences are all 1; in the diagonal that begins 1, 3, 6, …, the successive differences are 2, 3, 4, and so on. Do this up through the diagonal that begins 1, 6, 21, ….
   Student C: Find the values of the first five powers of the number 11, starting with 11^0 (recall $11^0 = 1$).
   Student D: Arrange the nine rows of the triangle with all rows "flush left," and then draw lightly dashed arrows as shown:



   and so on. Then add along the diagonals. Write these sums in order from left to right.
4. After all students have concluded their individual investigations in Item 3, return to a group discussion.
   (a) Have student A report the result found in Item 3, and then make a prediction concerning the sum of the entries in the tenth row.
   (b) Have student B report the successive differences discovered in the diagonals. Then have all students in the group investigate the successive differences in the diagonal that begins 1, 7, 28…. (It may be necessary to write a few more rows of the triangle.)
   (c) Have student C report the relationship between the powers of 11 found, and then determine the value of 11^5. Why does the pattern not continue here?
   (d) Have student D report the sequence of numbers found. Then, as a group, predict what the next sum will be by observing the pattern in the sequence. Confirm your prediction by actual computation.
5. Choose a representative from each group to report to the entire class the observations made throughout this investigation.
6. Find a reference to Pascal's Triangle using a search engine of the Internet and prepare a report on the reference.