Mathematical writing takes many forms. One of the most famous author/mathematicians was Charles Dodgson (1832–1898), who used the pen name Lewis Carroll.

Dodgson was a mathematics lecturer at Oxford University in England. Queen Victoria told Dodgson how much she enjoyed *Alice's Adventures in Wonderland* and how much she wanted to read his next book; he is said to have sent her *Symbolic Logic,* his most famous mathematical work.

The *Alice* books made Carroll famous. Late in life, however, Dodgson shunned attention and denied that he and Carroll were the same person, even though he gave away hundreds of signed copies to children and children's hospitals.

Using Writing to Learn About Mathematics

The use of writing in the mathematics curriculum has recently received increased attention, due in large part to the recommendations of the National Council of Teachers of Mathematics (N.C.T.M.). Research has indicated that the ability to express mathematical observations in writing can serve as a positive force in one's continued development as a mathematics student. The implementation of writing in the mathematics class can utilize several approaches.

Journals One way of using writing in mathematics is to keep a journal in which you spend a few minutes explaining what happened in class that day. The journal entries may be general or specific, depending on the topic covered, the degree to which you understand the topic, your interest level at the time, and so on. Journal entries are usually written in informal language, and are often an effective means of communicating to yourself, your classmates, and your instructor what feelings, perceptions, and concerns you are having at the time.

Learning Logs While journal entries are for the most part unstructured writings in which the student's thoughts are allowed to roam freely, entries in learning logs are typically more structured. An instructor may pose a specific question for a student to answer in a learning log. In this text, we intersperse writing exercises in each exercise set that are appropriate for answering in a learning log. For example, consider Exercise 13 in the exercise set for the opening section in this chapter.

> Discuss the differences between inductive and deductive reasoning. Give an example of each.

Here is a possible response to this exercise.

> Deductive reasoning occurs when you go from general ideas to specific ones. For example, I know that I can multiply both sides of $\frac{1}{2}x = 6$ by 2 to get $x = 12$, because I can multiply both sides of any equation by whatever I want (except 0). Inductive reasoning goes the other way. If I make a general conclusion from specific observations, that's inductive reasoning. Example - in the numbers 4, 8, 12, 16, and so on, I can conclude that the next number is 20, since I always add 4 to get the next number.

Reports on Articles from Mathematics Publications The motto "Publish or perish" has long been around, implying that a scholar in pursuit of an

academic position must publish in a journal in his or her field. There are numerous journals that publish papers in mathematics research and/or mathematics education. In Activity 3 at the end of this section, we provide some suggestions of articles that have appeared within the last few years. A report on such an article can help you understand what mathematicians do and what ideas mathematics teachers use to convey concepts to their students.

Term Papers   A term paper in a mathematics class? In increasing numbers, professors in mathematics survey courses are requiring short term papers of their students. In this way, you can become aware of the plethora of books and articles on mathematics and mathematicians, many written specifically for the layperson. In Activities 5 and 6 at the end of this section, we provide a list of possible term paper topics.

EXTENSION   ACTIVITIES

Rather than include a typical exercise set, we list some suggested activities in which writing can be used to enhance awareness and learning of mathematics.

Activity 1   Keep a journal. After each class, write for a few minutes on your perceptions about the class, the topics covered, or whatever you feel is appropriate. You may wish to use the following guidelines.

Journal Writing*

1. *WHO should write in your journal?*   You should.
2. *WHAT should you write in your journal?*   New words, ideas, formulas, or concepts; profound thoughts; wonderings, musings, problems to solve; reflections on the class; questions—both answerable and unanswerable; writing ideas
3. *WHEN should you write in your journal?*   After class each day; as you are preparing, reading, or studying for class; anytime an insight or question hits you.
4. *WHERE should you write in your journal?*   Anywhere—so keep it with you when possible.
5. *WHY should you write in your journal?*   It will help you record ideas that you might otherwise forget. It will be worthwhile for you to read later on so that you can note your growth. It will facilitate your learning, problem solving, writing, reading, and discussion in class.
6. *HOW should you write in your journal?*   In wonderful, long, flowing sentences with perfect punctuation and perfect spelling and in perfect handwriting; or in single words that express your ideas, in short phrases, in sketches, in numbers, in maps, in diagrams, in sentences (You may even prefer to organize your journal entries on your desktop, notebook, or palmtop computer.)

*"Journal Writing" from "No Time for Writing in Your Class?" by Margaret E. McIntosh in *Mathematics Teacher,* September 1991, p. 431. Reprinted by permission.

Activity 2 Keep a learning log, answering at least one writing exercise from each exercise set covered in your class syllabus. Ask your teacher for suggestions of other types of specific writing assignments. For example, you might wish to choose a numbered example from a section in the text and write your own solution to the problem, or comment on the method that the authors use to solve the problem. Don't be afraid to be critical of the method used in the text.

Activity 3 The National Council of Teachers of Mathematics publishes journals in mathematics education: *Teaching Children Mathematics* (formerly called *Arithmetic Teacher*) and *Mathematics Teacher* are two of them. These journals can be found in the periodicals section of most college and university libraries. We have chosen several recent articles in each of these journals. There are thousands of other articles from which to choose. Write a short report on one of these articles according to guidelines specified by your instructor.

From *Mathematics Teacher*

1998

Andreason, Corey. "Fibonacci and Pascal Together Again: Pattern Exploration in the Fibonacci Sequence." Vol. 91, No. 3, March 1998, p. 250.

Grassl, Richard M., and Tabitha T. Y. Mingus. "Keep Counting Those Boxes—There's More." Vol. 91, No. 2, February 1998, p. 122.

Iovanelli, Robert. "Using Spreadsheets to Analyze the Historical Perspectives of Apportionment." Vol. 91, No. 2, February 1998, p. 176.

Rothbart, Andrea. "Learning to Reason from Lewis Carroll." Vol. 91, No. 1, January 1998, p. 6.

Rulf, Benjamin. "A Geometric Puzzle That Leads to Fibonacci Sequences." Vol. 91, No. 1, January 1998, p. 21.

1999

Kahan, Jeremy. "Ten Lessons from the Proof of Fermat's Last Theorem." Vol. 92, No. 6, September 1999, p. 530.

Silver, Jennifer Williams. "A Survey on the Use of Writing-to-Learn in Mathematics Classes." Vol. 92, No. 5, May 1999, p. 388.

Smith, John P. III. "Preparing Students for Modern Work: Lessons from Automobile Manufacturing." Vol. 92, No. 3, March 1999, p. 254.

2000

Kelley, Loretta. "A Mathematical History Tour." Vol. 93, No. 1, January 2000, p. 14.

Lesser, Lawrence Mark. "Sum of Songs: Making Mathematics Less Monotone!" Vol. 93, No. 5, May 2000, p. 372.

Lightner, James E. "Mathematicians Are Human Too." Vol. 93, No. 8, November 2000, p. 696.

Natsoulas, Anthula. "Group Symmetries Connect Art and History with Mathematics." Vol. 93, No. 5, May 2000, p. 364.

Simon, Marilyn K. "The Evolving Role of Women in Mathematics." Vol. 93, No. 9, December 2000, p. 782.

2001

Johnson, Craig M. "Functions of Number Theory in Music." Vol. 94, No. 8, November 2001, p. 700.

Lightner, James E. "Mathematics Didn't Just Happen." Vol. 94, No. 9, December 2001, p. 780.

McNeill, Sheila A. "The Mayan Zeros." Vol. 94, No. 7, October 2001, p. 590.

Socha, Susan. "Less Is Sometimes More." Vol. 94, No. 6, September 2001, p. 450.

2002

Houser, Don. "Roots in Music." Vol. 95, No. 1, January 2002, p. 16.

Howe, Roger. "Hermione Granger's Solution." Vol. 95, No. 2, February 2002, p. 86.

Kolpas, Sidney J. "Let Your Fingers Do the Multiplying." Vol. 95, No. 4, April 2002, p. 246.

Van Dresar, Vickie J. "Opening Young Minds to Closure Properties." Vol. 95, No. 5, May 2002, p. 326.

From *Teaching Children Mathematics*

1998

Boucher, Alfred C. "Critical Thinking through Estimation." Vol. 4, No. 7, March 1998, p. 452.

Carey, Linda M. "Parents as Math Partners: A Successful Urban Story." Vol. 4, No. 6, February 1998, p. 314.

Fennell, Francis (Skip). "Mathematics at the Mall." Vol. 4, No. 5, January 1998, p. 268.

Olson, Melfried, Lynae Sakshag, and Judith Olson. "How Many Sandwiches?" Vol. 4, No. 7, March 1998, p. 402.

Zaslavsky, Claudia. "Ethnomathematics and Multicultural Mathematics Education." Vol. 4, No. 9, May 1998, p. 502.

1999

Angerame, Shirlee S. "Math-o'-Lanterns." Vol. 5, No. 11, October 1999, p. 72.

Basile, Carol G. "Collecting Data Outdoors: Making Connections to the Real World." Vol. 5, No. 10, September 1999, p. 8.

Brahier, Daniel J., and Melfried Olsen. "The World's Largest Math Event: Promoting Mathematical Thinking." Vol. 5, No. 7, March 1999, p. 430.

Sawada, Daiyo. "Mathematics as Problem Solving: A Japanese Way." Vol. 5, No. 7, September 1999, p. 54.

2000

Glasgow, Robert, et. al. "The Decimal Dilemma." Vol. 7, No. 2, October 2000, p. 89.

Hellwig, Stacey J., Eula Ewing Monroe, and James S. Jacobs. "Making Informed Choices: Selecting Children's Trade Books for Mathematics Instruction." Vol. 7, No. 3, November 2000, p. 138.

Lemme, Barbara. "Integrating Measurement Projects: Sand Timers." Vol. 7, No. 3, November 2000, p. 132.

Riddle, Margaret, and Bette Rodzwell. "Fractions: What Happens Between Kindergarten and the Army?" Vol. 7, No. 4, December 2000, p. 202.

2001

Karp, Karen S., and E. Todd Brown. "Geo-Dolls: Traveling in a Mathematical World." Vol. 8, No. 3, November 2001, p. 132.

Randolph, Tamela D., and Helene J. Sherman. "Alternative Algorithms: Increasing Options, Reducing Errors" Vol. 7, No. 8, April 2001, p. 480.

Sun, Wei, and Joanne Y. Zhang. "Teaching Addition and Subtraction Facts: A Chinese Perspective." Vol. 8, No. 1, September 2001, p. 28.

Whitenack, Joy W., et. al. "Second Graders Circumvent Addition and Subtraction Difficulties." Vol. 8, No. 4, December 2001, p. 228.

2002

Agosto, Melinda. "Cool Mathematics for Kids." Vol. 8, No. 7, March 2002, p. 397.

Huniker, DeAnn. "Calculators as Learning Tools for Young Children's Explorations of Number." Vol. 8, No. 6, February 2002, p. 316.

Strutchens, Marilyn E. "Multicultural Literature as a Context for Problem Solving: Children and Parents Learning Together." Vol. 8, No. 8, April 2002, p. 448.

Whitin, David J. "The Potentials and Pitfalls of Integrating Literature into the Mathematics Program." Vol. 8, No. 9, May 2002, p. 503.

Activity 4   One of the most popular mathematical films of all time is *Donald in Mathmagicland,* produced by Disney in 1959. It is available on video and may be rented at many video stores or purchased at outlets that sell video releases. Spend an entertaining half hour watching this film, and write a report on it according to the guidelines of your instructor.

Activity 5   Write a report according to the guidelines of your instructor on one of the following mathematicians, philosophers, and scientists.

Abel, N.	Cardano, G.	Gauss, C.	Noether, E.
Agnesi, M. G.	Copernicus, N.	Hilbert, D.	Pascal, B.
Agnesi, M. T.	De Morgan, A.	Kepler, J.	Plato
Al-Khowârizmi	Descartes, R.	Kronecker, L.	Polya, G.
Apollonius	Euler, L.	Lagrange, J.	Pythagoras
Archimedes	Fermat, P.	Leibniz, G.	Ramanujan, S.
Aristotle	Fibonacci	L'Hospital, G.	Riemann, G.
Babbage, C.	(Leonardo	Lobachevsky, N.	Russell, B.
Bernoulli, Jakob	of Pisa)	Mandelbrot, B.	Somerville, M.
Bernoulli,	Galileo (Galileo	Napier, J.	Tartaglia, N.
Johann	Galilei)	Nash, J.	Whitehead, A.
Cantor, G.	Galois, E.	Newton, I.	Wiles, A.

Activity 6   Write a term paper on one of the following topics in mathematics according to the guidelines of your instructor.

Babylonian mathematics	Pascal's triangle
Egyptian mathematics	The origins of probability theory
The origin of zero	Women in mathematics
Plimpton 322	Mathematical paradoxes
The Rhind papyrus	Unsolved problems in
Origins of the Pythagorean	mathematics
Theorem	The four-color theorem
The regular (Platonic) solids	The proof of Fermat's Last
The Pythagorean brotherhood	Theorem
The Golden Ratio (Golden	The search for large primes
Section)	Fractal geometry
The three famous construction	The co-inventors of calculus
problems of the Greeks	The role of the computer in the
The history of the approximations	study of mathematics
of π	Mathematics and music
Euclid and his "Elements"	Police mathematics
Early Chinese mathematics	The origins of complex numbers
Early Hindu mathematics	Goldbach's conjecture
Origin of the word *algebra*	The use of the Internet in
Magic squares	mathematics education
Figurate numbers	The development of graphing
The Fibonacci sequence	calculators
The Cardano/Tartaglia controversy	Mathematics education reform
Historical methods of computation	movement
(logarithms, the abacus, Napier's	Multicultural mathematics
rods, the slide rule, etc.)	

Activity 7   Investigate a computer program that focuses on teaching children elementary mathematics and write a critical review of it as if you were writing for a journal that contains software reviews of educational material. Be sure to address the higher-level thinking skills in addition to drill and practice.

Activity 8   The following Websites provide a fascinating list of mathematics-related topics. Go to one of them, choose a topic that interests you, and report on it, according to the guidelines of your instructor.

http://mathworld.wolfram.com/

http://world.std.com/~reinhold/mathmovies.html

http://www.mcs.surrey.ac.uk/Personal/R.Knott/

http://dir.yahoo.com/Science/Mathematics/

http://www.cut-the-knot.com/

http://www.agnesscott.edu/lriddle/women/women.htm

http://www.ics.uci.edu/~eppstein/recmath.html