## 4.3 EXERCISES

List the first twenty counting numbers in each of the following bases.

- **1.** seven (Only digits 0 through 6 are used in base seven.)
- 3. nine (Only digits 0 through 8 are used.)

- **2.** eight (Only digits 0 through 7 are used.)
- **4.** sixteen (The digits 0, 1, 2, ..., 9, A, B, C, D, E, F are used in base sixteen.)

For each of the following, write (in the same base) the counting numbers just before and just after the given number. (Do not convert to base ten.)

**5.** 14<sub>five</sub> **6.** 555<sub>six</sub> **7.** B6F<sub>sixteen</sub> **8.** 10111<sub>two</sub>

Determine the number of distinct symbols needed in each of the following positional systems.

9. base three 10. base seven 11. base eleven 12. base sixteen

Determine, in each of the following bases, the smallest and largest four-digit numbers and their decimal equivalents. 13. three 14. sixteen

 $01_{two}$ 

Convert each of the following to decimal form by expanding in powers and by using the calculator shortcut.

<b>15.</b> 24 <sub>five</sub>	<b>16.</b> 62 <sub>seven</sub>	<b>17.</b> 1011 <sub>two</sub>	<b>18.</b> 35 <sub>eight</sub>
<b>19.</b> 3BC <sub>sixteen</sub>	<b>20.</b> 34432 <sub>five</sub>	<b>21.</b> 2366 <sub>seven</sub>	<b>22.</b> 101101110 <sub>two</sub>
<b>23.</b> 70266 <sub>eight</sub>	24. ABCD <sub>sixteen</sub>	<b>25.</b> 2023 <sub>four</sub>	<b>26.</b> 6185 <sub>nine</sub>
<b>27.</b> 41533 <sub>six</sub>	<b>28.</b> 88703 nine		

Convert each of the following from decimal form to the given base.

<b>29.</b> 86 to base five	<b>30.</b> 65 to base seven	<b>31.</b> 19 to base two
<b>32.</b> 935 to base eight	<b>33.</b> 147 to base sixteen	<b>34.</b> 2730 to base sixteen
<b>35.</b> 36401 to base five	<b>36.</b> 70893 to base seven	<b>37.</b> 586 to base two
<b>38.</b> 12888 to base eight	<b>39.</b> 8407 to base three	<b>40.</b> 11028 to base four
<b>41.</b> 9346 to base six	<b>42.</b> 99999 to base nine	

Make the following conversions as indicated.

43.	$43_{\text{five}}$ to base seven	44.	$27_{eight}$ to base five
45.	6748 <sub>nine</sub> to base four	46.	$C02_{sixteen}$ to base seven

Convert each of the following from octal form to binary form.

<b>47.</b> 367 <sub>eight</sub>	<b>48.</b> 2406 <sub>eight</sub>
-7. 507 <sub>eight</sub>	<b>40.</b> 2400 <sub>eight</sub>

Convert each of the following from binary form to octal form.

<b>49.</b> 100110111 <sub>two</sub> <b>50.</b> 1101	01111
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Make the following conversions as indicated.

51.	DC <sub>sixteen</sub> to binary	<b>52.</b> $F111_{sixteen}$ to binary
53.	101101 <sub>two</sub> to hexadecimal	<b>54.</b> $101111011101000_{two}$ to hexadecimal

Identify the largest number from each list in Exercises 55 and 56.

<b>55.</b> 42 <sub>seven</sub> , 37 <sub>eight</sub> , 1D <sub>sixteen</sub>	<b>56.</b> 1101110 <sub>two</sub> , 407 <sub>five</sub> , 6F <sub>sixteen</sub>
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There is a theory that twelve would be a better base than ten for general use. This is mainly because twelve has more divisors (1, 2, 3, 4, 6, 12) than ten (1, 2, 5, 10), which makes fractions easier in base twelve. The base twelve system is called the **duodecimal system**. Just as in the decimal system we speak of a one, a ten, and a hundred (and so on), in the duodecimal system we say a one, a dozen (twelve), and a gross (twelve squared, or one hundred forty-four).

57. Otis Taylor's clients ordered 9 gross, 10 dozen, and 11 copies of *The Minnie Minoso Story* during 2002. How many copies was that in base ten?
58. Which amount is larger: 3 gross, 6 dozen or 2 gross, 19 dozen?

One very common method of converting symbols into binary digits for computer processing is called ASCII (American Standard Code of Information Interchange). The uppercase letters A through Z are assigned the numbers 65 through 90, so A has binary code 1000001 and Z has code 1011010. Lowercase letters a through z have codes 97 through 122 (that is, 1100001 through 1111010). ASCII codes, as well as other numerical computer output, normally appear without commas.

Write the binary code for each of the following letters.

**59.** C **60.** X **61.** k

Break each of the following into groups of seven digits and write as letters.

**62.** 100100010010110011001010000**63.** 100001110010001010101010000111001011

Translate each word into an ASCII string of binary digits. (Be sure to distinguish upper and lowercase letters.)

64. New

**66.** Explain why the octal and hexadecimal systems are convenient for people who code for computers.

**65.** Orleans

**67.** There are thirty-seven counting numbers whose base eight numerals contain two digits but whose base three numerals contain four digits. Find the smallest and largest of these numbers.

Refer to Table 9 for Exercises 68–71.

**68.** After observing the binary forms of the numbers 1-31, identify a common property of all Table 9 numbers in each of the following columns.

(a) Column A (b) Column B (c) Column C (d) Column D (e) Column E

**69.** Explain how the "trick" of Table 9 works.

70. How many columns would be needed for Table 9 to include all ages up to 63?

71. How many columns would be needed for Table 9 to include all numbers up to 127?

In our decimal system, we distinguish odd and even numbers by looking at their ones (or units) digits. If the ones digit is even (0, 2, 4, 6, or 8), the number is even. If the ones digit is odd (1, 3, 5, 7, or 9), the number is odd. For Exercises 72–79, determine whether this same criterion works for numbers expressed in the given bases.

72.	two 73.	three	<b>74.</b> four	<b>75.</b> five
76.	six 77.	seven	<b>78.</b> eight	<b>79.</b> nine
<sup>⊗</sup> 80.	Consider all even bases. If the explain why. If not, find one even bases.	criterion works for all, $\Im$ that does work for all	<b>1.</b> Consider all odd bases. If explain why. If not, find codd bases.	the criterion works for all, one that does work for all

Determine whether the given base five numeral represents one that is divisible by five.

<b>82.</b> 3204 <sub>five</sub>	<b>83.</b> 200 <sub>five</sub>	<b>84.</b> 2310 <sub>five</sub>	<b>85.</b> 342 <sub>five</sub>