4.4 EXERCISES

For each system in Exercises 1–10, decide which of the properties of single-operation systems are satisfied. If the identity property is satisfied, give the identity element. If the inverse property is satisfied, give the inverse of each element. If the identity property is satisfied but the inverse property is not, name the elements that have no inverses.

1. $\{1, 2\}$; operation $\otimes$

$\otimes$	**1**	**2**
1	1	2
2	2	1

2. $\{1, 2, 3, 4\}$; operation $\otimes$

$\otimes$	**1**	**2**	**3**	**4**
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

3. $\{1, 2, 3, 4, 5, 6, 7\}$; operation $\boxtimes$

$\boxtimes$	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1	1	2	3	4	5	6	7
2	2	4	6	0	2	4	6
3	3	6	1	4	7	2	5
4	4	0	4	0	4	0	4
5	5	2	7	4	1	6	3
6	6	4	2	0	6	4	2
7	7	6	5	4	3	2	1

4. $\{1, 2, 3, 4, 5\}$; operation $\boxtimes$

$\boxtimes$	**1**	**2**	**3**	**4**	**5**
1	1	2	3	4	5
2	2	4	0	2	4
3	3	0	3	0	3
4	4	2	0	4	2
5	5	4	3	2	1

5. $\{1, 3, 5, 7, 9\}$; operation ☆

☆	**1**	**3**	**5**	**7**	**9**
1	1	3	5	7	9
3	3	9	5	1	7
5	5	5	5	5	5
7	7	1	5	9	3
9	9	7	5	3	1

6. $\{1, 3, 5, 7\}$; operation ☆

☆	**1**	**3**	**5**	**7**
1	1	3	5	7
3	3	1	7	5
5	5	7	1	3
7	7	5	3	1

7. $\{A, B, F\}$; operation *

*	***A***	***B***	***F***
A	*B*	*F*	*A*
B	*F*	*A*	*B*
F	*A*	*B*	*F*

8. $\{m, n, p\}$; operation ***J***

J	***m***	***n***	***p***
m	*n*	*p*	*n*
n	*p*	*m*	*n*
p	*n*	*n*	*m*

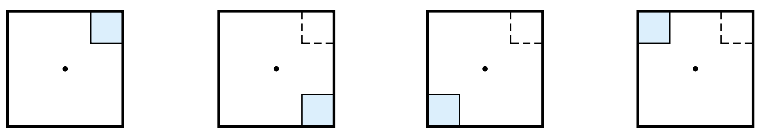
9. $\{r, s, t, u\}$; operation ***Z***

Z	***r***	***s***	***t***	***u***
r	*u*	*t*	*r*	*s*
s	*t*	*u*	*s*	*r*
t	*r*	*s*	*t*	*u*
u	*s*	*r*	*u*	*t*

10. $\{A, J, T, U\}$; operation #

#	***A***	***J***	***T***	***U***
A	*A*	*J*	*T*	*U*
J	*J*	*T*	*U*	*A*
T	*T*	*U*	*A*	*J*
U	*U*	*A*	*J*	*T*

The tables in the finite mathematical systems that we developed in this section can be obtained in a variety of ways. For example, let us begin with a square, as shown in the figure. Let the symbols a, b, c, and d be defined as shown in the figure.



Let *a* represent zero rotation—leave the original square as is.

Let *b* represent rotation of 90° clockwise from original position.

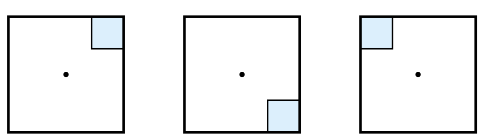
Let *c* represent rotation of 180° clockwise from original position.

Let *d* represent rotation of 270° clockwise from original position.

Define an operation □ for these letters as follows. To evaluate b □ c, for example, first perform b by rotating the square 90°. *(See the figure.) Then perform operation c by rotating the square an additional* 180°. *The net result is the same as if we had performed d only. Thus,*

$$b \,\square\, c = d.$$

Start with *a*. Perform *b*. Start with *b*, and perform *c*.



Use this method to find each of the following.

11. $b \,\square\, d$ **12.** $b \,\square\, b$ **13.** $d \,\square\, b$ **14.** $a \,\square\, b$

Solve each problem.

15. Complete the table at the right.

$\square$	a	b	c	d
a	a	b	c	d
b	b	c		a
c	c		a	
d	d	a		

16. Which of the properties from this section are satisfied by this system?

17. Define a universal set U as the set of counting numbers. Form a new set that contains all possible subsets of U. This new set of subsets together with the operation of set intersection forms a mathematical system. Which of the properties listed in this section are satisfied by this system?

18. Replace the word "intersection" with the word "union" in Exercise 17; then answer the same question.

19. Complete the table at the right so that the result is *not* the same as operation $\square$ of Exercise 15, but so that the five properties listed in this section still hold.

	a	b	c	d
a				
b				
c				
d				

Try examples to help you decide whether the following operations, when applied to the integers, satisfy the distributive property.

20. subtraction with respect to multiplication

21. addition with respect to subtraction

22. subtraction with respect to addition

Recall that Example 3 provided a counterexample for the general statement

$$a + (b \times c) = (a + b) \times (a + c).$$

Thus, addition is not *distributive with respect to multiplication. Now work Exercises 23–26.*

23. Decide if the statement above is true for each of the following sets of values.
(a) $a = 2, b = -5, c = 4$
(b) $a = -7, b = 5, c = 3$
(c) $a = -8, b = 14, c = -5$
(d) $a = 1, b = 6, c = -6$

24. Find another set of a, b, and c values that make the statement true.

25. Under what general conditions will the statement above be true?

26. Explain why, regardless of the results in Exercises 23–25, addition is still *not* distributive with respect to multiplication.

27. Give the conditions under which each of the following equations would be true.
(a) $a + (b - c) = (a + b) - (a + c)$
(b) $a - (b + c) = (a - b) + (a - c)$

28. **(a)** Find values of a, b, and c such that

$$a - (b \times c) = (a - b) \times (a - c).$$

(b) Does this mean that subtraction is distributive with respect to multiplication? Explain.

Verify for the mathematical system of Example 4 that the distributive property holds for the following cases.

29. $c \star (d \circ e) = (c \star d) \circ (c \star e)$

30. $a \star (a \circ b) = (a \star a) \circ (a \star b)$

31. $d \star (e \circ c) = (d \star e) \circ (d \star c)$

32. $b \star (b \circ b) = (b \star b) \circ (b \star b)$

These exercises are for students who have studied sets.

33. Use Venn diagrams to show that the distributive property for union with respect to intersection holds for sets A, B, and C. That is,

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

34. Use Venn diagrams to show that *another* distributive property holds for sets A, B, and C. It is the distributive property of intersection with respect to union.

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

These exercises are for students who have studied logic.

35. Use truth tables to show that the following distributive property holds:

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r).$$

36. Use truth tables to show that *another* distributive property holds:

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r).$$