4.5 EXERCISES

What is wrong with the way in which each of the following questions is stated?

1. Do the integers form a group?

2. Does multiplication satisfy all of the group properties?

Decide whether each system is a group. If not a group, identify all properties that are not satisfied. (Recall that any system failing to satisfy the identity property automatically fails to satisfy the inverse property also.) For the finite systems, it may help to construct tables. For infinite systems, try some examples to help you decide.

3. $\{0\}$; multiplication

4. $\{0\}$; addition

5. $\{0, 1\}$; addition

6. $\{0\}$; subtraction

7. $\{-1, 1\}$; division

8. $\{0, 1\}$; multiplication

9. $\{-1, 0, 1\}$; multiplication

10. $\{-1, 0, 1\}$; addition

11. integers; subtraction

12. integers; multiplication

13. odd integers; multiplication

14. counting numbers; addition

15. rational numbers; addition

16. even integers; addition

17. prime numbers; addition

18. nonzero rational numbers; multiplication

19. Explain why a *finite* group based on the operation of ordinary addition of numbers cannot contain the element 1.

20. Explain why a group based on the operation of ordinary addition of numbers *must* contain the element 0.

Exercises 21–34 apply to the system of symmetries of a square presented in the text.

Find each of the following.

21. RN **22.** PR **23.** TV **24.** VP

Verify each of the following statements.

25. $N(TR) = (NT)R$ **26.** $V(PS) = (VP)S$ **27.** $T(VN) = (TV)N$ **28.** $S(MR) = (SM)R$

Find the inverse of each element.

29. N **30.** Q **31.** R **32.** S **33.** T **34.** V

A group that also satisfies the commutative property is called a commutative group *(or an* abelian group, *after Niels Henrik Abel).*

Determine whether each of the following groups is commutative.

35. the group of symmetries of a square

36. the subgroup of Example 6

37. the integers under addition

38. the permutation group on three symbols

Give illustrations to support your answers for Exercises 39–42.

39. Produce a mathematical system with two operations which is a group under one operation but not a group under the other operation.

40. Explain what property is gained when the system of counting numbers is extended to the system of whole numbers.

41. Explain what property is gained when the system of whole numbers is extended to the system of integers.

42. Explain what property is gained when the system of integers is extended to the system of rational numbers.