chapter

4 Numeration and Mathematical Systems

The earliest episodes from the history of mathematics make it clear that the need for counting was the basis for the development of numeration systems. As you will see in this chapter, these systems range in concept from the simplest ones, which use tally marks, to our current place value system. One of the greatest advantages of our current system is that algorithms, or procedures, for computation are relatively easy. For example, when we add or subtract two numbers in a vertical manner, we "line them up in columns" so that digits representing the same place values are combined. It is possible that this procedure is the only one you have ever used when adding and subtracting using pencil and paper. There are, however, alternative methods for performing computations, and some of them are covered in this chapter.

One of the most interesting of these alternative algorithms is the *nines complement method* for subtracting. To use this method, we first agree that the nines complement of a digit n is $9 - n$. For example, the nines complement of 0 is 9, of 1 is 8, of 2 is 7, and so on, up to the nines complement of 9, which is 0. Now suppose we want to subtract

$$\begin{array}{r} 25{,}341 \\ -\,8{,}496. \\ \hline \end{array}$$

We fill in any missing place values in the subtrahend (8,496) with zeros so that there are the same number of digits in both numbers:

$$\begin{array}{r} 25{,}341 \\ -\,08{,}496. \\ \hline \end{array}$$

Next, replace each digit in the subtrahend with its nines complement, and then add:

$$\begin{array}{r} 25{,}341 \\ +\,91{,}503 \\ \hline 116{,}844. \end{array}$$

Now, delete the first digit (1), and add 1 to the remaining part of the sum (16,844):

$$16{,}844 + 1 = \boxed{16{,}845}. \leftarrow \text{Answer}$$

The answer to the original subtraction problem is 16,845.

Earlier we introduced and studied the concept of a *set*, a collection of elements. A set, in itself, may have no particular structure. But when we introduce *ways of combining the elements* (called *operations*) and *ways of comparing the elements* (called *relations*), we obtain a **mathematical system.**

Mathematical System

A **mathematical system** is made up of three components:

1. a set of elements;
2. one or more operations for combining the elements;
3. one or more relations for comparing the elements.

Symbols designed to represent objects or ideas are among the oldest inventions of humans. These Indian symbols in Arizona are several hundred years old.

A familiar example of a mathematical system is the set of whole numbers $\{0, 1, 2, 3, \ldots\}$, along with the operation of addition and the relation of equality.

Historically, the earliest mathematical system to be developed involved the set of counting numbers or initially a limited subset of the "smaller" counting numbers. The various ways of symbolizing and working with the counting numbers are called **numeration systems.** The symbols of a numeration system are called **numerals.**