3.3.23 (corrected). In addition to hypotheses (i) and (ii) of exercise 21 for P, also assume that (iii) for every vertex V in P the length of a shortest path from S to V is less than or equal to the length of a shortest path from S to any other vertex W not in P. (In other words P contains the vertices closest to S.) Let $P' = P \cup \{U\}$. Show that P' satisifies properties (i) and (iii). Show that for all $V \notin P'$, the length of a shortest path from S to V, all of whose vertices except V are in P', is the minimum of $L(V)$ and $L(U) + W(U, V)$.

Hints for 3.3.21.

1. Notice that the exercise says nothing about Dijkstra's algorithm. Don't assume that the set P was obtained by running Dijkstra's algorithm on G.
2. You know there is a path from S to U whose only vertex not in P is U and whose length is $L(U)$. Now suppose there is a strictly shorter path from S to U which contains some other vertex not in P on the way. Pick the first vertex X along this path that is not in P and show that $L(X) < L(U)$. This will contradict that U was chosen with minimal label among the vertices not in P.

Hint for 3.3.22. You showed in 21 that a shortest path from S to U can be chosen so that its vertices are all in P except for U itself. Why is the length of this path $L(U)$?

Hint for 3.3.24. Use the initial setup of P and L from Dijkstra's algorithm as the initial step of the induction, and show that it satisfies conditions (i), (ii), and (iii) from exercise 21 and 23. Now use the result you proved in 23.