The condition in Theorem 3.6 in the text, whose proof we completed in class today, says that any two nonadjacent vertices x and y has

$$\deg(x) + \deg(y) \geq \text{the number of vertices of } G.$$

In the proof, we picked two nonadjacent vertices x and y, showed that there is a Hamiltonian path from x to y, and used the degree condition to construct a Hamiltonian circuit from this path. It would seem that we only use the degree condition for this particular x and y and that any two nonadjacent vertices could be chosen for x and y. Wouldn't it be enough to require that there exist two nonadjacent vertices x and y for which the degree condition holds? Or do we really need the degree condition to hold for any two nonadjacent vertices? Why?