Let G be an undirected graph. The line graph L(G) of G is the undirected graph whose vertices correspond to the edges of G, and two vertices are adjacent if and only if the corresponding edges in G are incident to the same vertex. See the graphs below for an example of this, where the correspondence is $e_i \leftrightarrow v_i$.



- (a) Prove that if G has an Euler circuit, then L(G) also has an Euler circuit. (Hint: consider the degrees of the vertices in G and L(G).
- (b) Prove that the converse is not true, that is if L(G) has an Euler circuit, G need not have an Euler circuit. (Hint: you can prove something is not true by finding a counterexample.)