Let T be a finite, simple, undirected graph of at least 2 vertices. Prove by induction that if T has one more vertex than edges and has no cycles then T is a tree.

Hints:

1. Induct on the number of vertices. Base case is $n = 2$.
2. Show that T has either a vertex of degree 1 or an isolated vertex. If there is an isolated vertex, remove it and any one edge and use the inductive hypothesis. Now add the edge back, but not the vertex and reach a contradiction. Conclude that there must be a vertex of degree 1.
3. Remove the vertex of degree 1 together with its incident edge and use the inductive hyopthesis.