

Math 524 Homework 1

Due at 5:30 PM on 9/13

1. Decide if the following are operations on the given sets. For those that are, say if they are commutative and/or associative. Is there an identity and if there is, what is it? Be sure to justify your answers.
 - (a) The cross product on \mathbb{R}^3 .
 - (b) $x \circ y = xy - 1$ on \mathbb{Z} ,
 - (c) $x \circ y = \sqrt{xy}$ on \mathbb{Q}^+ (the positive rational numbers),
 - (d) $x \circ y = x\overline{y}$ on \mathbb{C} ,
 - (e) $x \circ y = 2^{xy}$ on \mathbb{Z}^+ .
2. Let \circ be an operation on the set S and let $T \subseteq S$. Prove or give a counterexample:
 - (a) \circ is also an operation on T .
 - (b) If \circ is associative on S and is an operation on T , then it is associative on T .
 - (c) If \circ is commutative on S and is an operation on T , then it is commutative on T .
 - (d) If \circ has an identity on S and is an operation on T , then it has an identity on T .
 - (e) If \circ is an operation on T and has an identity e on T , then e is also an identity on S .
 - (f) If \circ is an operation on T , has an identity e on S and $e \in T$, then e is an identity on T .
3. Let \circ be an operation on S with identity e . Show that if

$$x \circ (y \circ z) = (x \circ z) \circ y \quad \forall x, y, z \in S$$

then \circ is commutative and associative. (Hint: Prove commutativity first.)