MCS 118 Exam 2

Nov 9, 2018

All of your answers must be carefully justified. Neat work, clear and to-the-point explanations will receive more credit than messy, chaotic answers. You may refer to any result proved in class unless otherwise specified. You may use results you proved on your homework, except for ones the problem specifically asks you to prove.

You are not allowed to use your textbook or your class notes, but you may use a simple calculator.

1. (10 pts) Let $f(x) = \frac{x}{1+x}$ and $g(x) = \sin(2x)$. Find $f \circ g(x)$ and the largest possible subset of the real numbers that could be the domain of $f \circ g$.

2. (10 pts) Sketch the graph of an example of a function f that satisfies all of the conditions
$$\lim_{x\to 0^-} f(x) = 2, \qquad \lim_{x\to 0^+} f(x) = 0, \qquad f(0) = 2,$$
$$\lim_{x\to 4^-} f(x) = 3, \qquad \lim_{x\to 4^+} f(x) = 0, \qquad f(4) = 1.$$
Be sure to explain how you came up with the graph.

3. (5 pts each) Suppose f is an even function.
   (a) If g is any other function and $h = f \circ g$, is h always an even function?
   (b) If g is any other function and $h = g \circ f$, is h always an even function?

4. (5 pts each)
   (a) Let f be a function and a and L real numbers. State the formal $(\delta - \epsilon)$ definition of
   $$\lim_{x\to a} f(x) = L.$$
   (b) Give an example of a function f and a number a such that some neighborhood of a (that is an open interval surrounding a) except possibly a itself lies within the domain of f, and $\lim_{x\to a} f(x)$ does not exist. Of course, you will have to explain why the limit does not exist. Note that you will have to specify the domain of f so that you can check that some open interval surrounding a is in fact contained in it.

5. (10 pts) **Extra credit problem.** Let $f : \mathbb{R}^* \to \mathbb{R}$ be any odd function (remember $\mathbb{R}^*$ is the set of nonzero real numbers). Prove that either $\lim_{x\to 0} f(x)$ does not exist or $\lim_{x\to 0} f(x) = 0$. (Hint: look at the left and right limits of f at 0.)