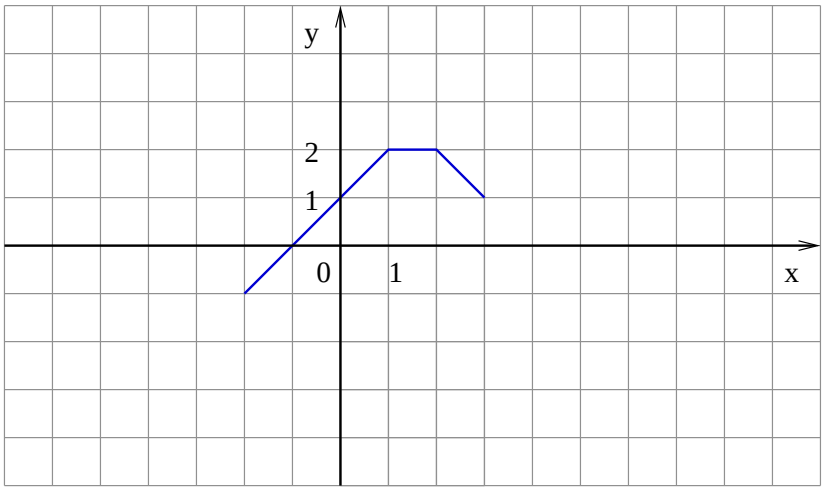
# MCS 118 Final Exam

Dec 19, 2018

**All of your answers must be carefully justified.** Neat work, clear and to-the-point explanations will receive more credit than messy, chaotic answers. You may refer to any result proved in class unless otherwise specified. You may use results you proved on your homework, except for ones the problem specifically asks you to prove.

You are not allowed to use your textbook or your class notes, but you may use a simple calculator.

1. (5 pts each) The graph of $f$ is given. Draw the graphs of the following functions. Feel free to use the same coordinate axes already given in the diagram, but be sure to explain what you did to the graph of $f$ to construct the new graph.
   (a) $g(x) = -2f(x)$
   (b) $h(x) = f\left(\frac{1}{3}x\right) + 1$




2. (10 pts) Prove the statement using the $\delta - \epsilon$ definition of a limit:

$$\lim_{x \to 0} x^3 = 0.$$

3. (10 pts) Use the Limit Laws to find the exact value of

$$\lim_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3}}{x}.$$

Do not forget to justify your work by referring to the Limit Laws and any other result you use.

4. (5 pts each)

   (a) Let $S$ and $T$ be nonempty sets. State the definition of a function $f$ from $S$ to $T$. What are the domain and the codomain of $f$?

   (b) Let $S$ be the set of all gifts in Santa's bag, including those lumps of coal for the naughty children. Let $T$ be the set of all elves that are employed at Santa's North Pole factory. Let $e : S \to T$ be the rule

   $$e(x) = \text{the elf that wrapped } x.$$

   Is $e$ a function?

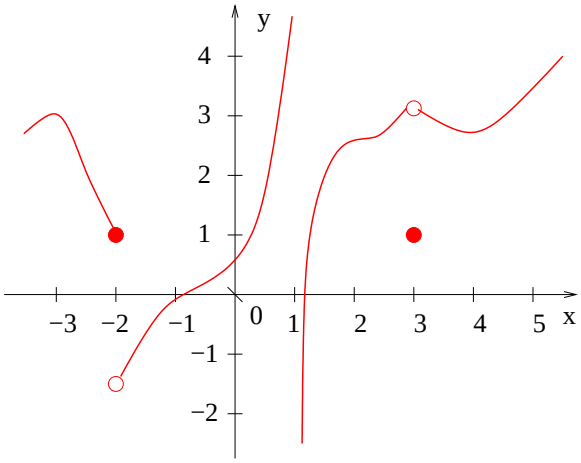
5. (5 pts each)

   (a) Give an example of a function that is neither even nor odd. Don't forget to justify your example, that is explain why it is neither even nor odd.

   (b) Give an example of a nonlinear function that is decreasing throughout its domain. Again, remember to justify your example.

6. (a) (4 pts) Define what a continuous function is.

   (b) (6 pts) At what numbers is the function $f$ whose graph is given below discontinuous? Identify what type of discontinuity $f$ has at each such number.




7. **Extra credit problem.**

   (a) (10 pts) We noted in class that

   $$\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$$

   does not exist. Use the $\delta - \epsilon$ definition of a limit to prove this. Hint: suppose the limit exists and is $L$, and show that for $\epsilon = 1/2$ there is no $\delta > 0$ that would satisfy the $\delta - \epsilon$ definition of a limit.

   (b) (5 pts) Would a similar argument show that

   $$\lim_{x \to 0} \cos\left(\frac{1}{x}\right)$$

   does not exist? Explain your reasoning.