MCS 118 Exam 1

Oct 14, 2019

All of your answers must be carefully justified. Neat work, clear and to-the-point explanations will receive more credit than messy, chaotic answers. You may refer to any result proved in class unless otherwise specified. You may use results you proved on your homework, except for ones the problem specifically asks you to prove.

You are not allowed to use your textbook or your class notes, but you may use a simple calculator.

1. (10 pts) Determine if the function $f : \mathbb{R} \to \mathbb{R}$ given by
$$f(x) = x|x|$$
is even, odd, or neither and justify your answer. If you have a graphing calculator, you can check your answer visually, or even get an idea of what the answer should be, but the apparent symmetry of a graph on the screen of a graphing calculator is not good evidence of evenness or oddness.

2. The monthly cost of driving a car depends on the number of miles driven. Lynn found that in May, it cost her \$380 to drive 480 miles and in June it cost her \$460 to drive 800 miles.
   (a) (4 pts) Express the monthly cost C as a function of the distance driven d assuming that a linear relationship gives a suitable model.
   (b) (4 pts) Draw the graph of the linear function. What does the slope represent? (Hint: what are the units of the slope?)
   (c) (2 pts) What does the vertical intercept represent?
3. (10 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be the function $f(x) = |3-x||x+1|$. Express f as a piecewise defined function. (Hint: you do not necessarily need to consider four cases here; you can get away with only two.)
4. (5 pts each)
   (a) Give an example of a polynomial $p(x)$ of degree 3 whose value is 0 at $x = -3$, $x = 2$, and $x = 5$ and whose value is 36 at $x = 3$.
   (b) Define what a rational function is and give an example of a rational function whose domain is all real numbers except $x = 4$. Do not forget to justify your example.
5. (10 pts) **Extra credit problem.** Let $f : \mathbb{R} \to \mathbb{R}$ be an increasing function such that $f(x) \neq 0$ for any real number x. Is the function $g(x) = f(-x)$ always decreasing? If you think it is, give a proof; if you do not think it is, find a counterexample.