MCS 118 EXAM 2 Nov 8, 2019

All of your answers must be carefully justified. Neat work, clear and to-the-point explanations will receive more credit than messy, chaotic answers. You may refer to any result proved in class unless otherwise specified. You may use results you proved on your homework, except for ones the problem specifically asks you to prove.

You are not allowed to use your textbook or your class notes, but you may use a simple calculator.

- 1. (10 pts) Graph the function $f(x) = x^2 + 6x + 4$ by hand, not by plotting points, but by starting with the graph of a standard power function and then applying appropriate transformations.
- 2. (5 pts each) Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt[3]{1-x}$.
 - (a) Find $g \circ f(x)$ and the largest possible subset of the real numbers that could be the domain of $g \circ f$.
 - (b) Find $f \circ g(x)$ and the largest possible subset of the real numbers that could be the domain of $f \circ g$.
- 3. (a) (4 pts) Let f be a function of real numbers and a and L real numbers. State the definition (informal or formal, your choice) of

$$\lim_{x \to a} f(x) = L.$$

- (b) (6 pts) Give an example of a function f and a number a such that $\lim_{x\to a} f(x)$ does not exist. Be sure to justify your example.
- 4. (10 pts) Let $f(x) = \frac{1}{x^2}$. Show that the value of f(x) can be made to be within 0.01 of 1/9 if x is sufficiently close to 3 by finding two numbers $x_1 < 3 < x_2$ such that if $x_1 < x < x_2$ then

$$\frac{1}{9} - 0.01 < f(x) < \frac{1}{9} + 0.01.$$

- 5. (5 pts each) **Extra credit problem.** Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be functions.
 - (a) Show that if f is increasing and $g : \mathbb{R} \to \mathbb{R}$ is decreasing then the composition $f \circ g$ is always a decreasing function. (Remember that an example does not prove that $f \circ g$ is always a decreasing function no matter what increasing function f is and what decreasing function g is.)
 - (b) What if f and g are both decreasing? Does $f \circ g$ have to increasing or decreasing or can it be neither in this case?