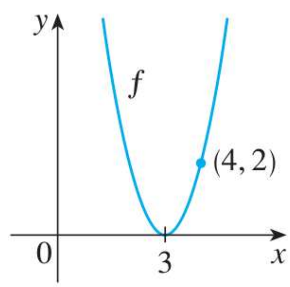
MCS 118 Final Exam

Dec 18, 2019

All of your answers must be carefully justified. Neat work, clear and to-the-point explanations will receive more credit than messy, chaotic answers. You may refer to any result proved in class unless otherwise specified. You may use results you proved on your homework, except for ones the problem specifically asks you to prove.

You are not allowed to use your textbook or your class notes, but you may use a simple calculator.

1. (10 pts) Find the expression for the quadratic function whose graph is shown. Be sure to carefully explain (in words) how you got your answer.



2. (10 pts) Prove the statement with the $\delta - \epsilon$ definition of a limit:

$$\lim_{x \to -2} (3x + 5) = -1.$$

3. (10 pts) Use the definition of continuity and the properties of limits to show that the function

$$f(x) = 3x^4 - 5x + \sqrt[3]{x^2 + 4}$$

is continuous at 2.

4. (5 pts each)
   (a) Define what even and odd functions are.
   (b) Let $f : \mathbb{R} \to \mathbb{R}$ be an even function and $g : \mathbb{R} \to \mathbb{R}$ be an odd function such that $g(x) \neq 0$. What kind of symmetry (even, odd, neither?) does f/g have in general? (Remember that one specific example is not sufficient evidence to prove a general statement.)

5. (10 pts) Find

$$\lim_{x \to 0^+} \left[\cos\left(\frac{1-x}{x}\right) \sin(x) \right].$$

(Hint: Refer to the illustration on the left. Use the Sandwich (Squeeze) Theorem.)

6. (5 pts each) Let

$$f(x) = \frac{x^2 + 2x - 8}{|x - 2|}.$$

   (a) Find

$$\lim_{x \to 2} f(x)$$

   if it exists. If the limit does not exist, explain why it does not exist.

(b) Find

$$\lim_{x \to -4} f(x)$$

if it exists. If the limit does not exist, explain why it does not exist.

7. **Extra credit problem.** Let f and g be functions and a a real number such that

$$\lim_{x \to a} f(x) = L_1 \qquad \text{and} \qquad \lim_{x \to a} g(x) = L_2$$

both exist.

(a) (10 pts) Prove that if $f(x) \leq g(x)$ for every x is some neighborhood of a except possibly at $x = a$, then $L_1 \leq L_2$. (Hint: Suppose that $L_1 \leq L_2$ is not true, that is $L_2 < L_1$. Now use the definition of the limit with $\epsilon = (L_1 - L_2)/2$ to show that it cannot be true that $f(x) \leq g(x)$ for every x near a.)

(b) (5 pts) Now suppose that $f(x) < g(x)$ for every x is some neighborhood of a except possibly at $x = a$. Does it have to be true that $L_1 < L_2$? If you think it is true, give a proof; if you do not think it is true, find a counterexample.