# Some advice on writing limits

Here is some advice I mostly stole for you from Prof. Siehler on how to write calculations with limits. If you heed this advice, you will communicate your work with limits more effectively and you will make fewer mistakes. That has a number of benefits for both you and me. For one, your calculations are more likely to be correct if your thinking and writing are more organized. Even if you make a mistake, I will have an easier time finding what went wrong and understanding why it went wrong, and also seeing what you did right. Think partial credit. In fact, if you end up with a final result that looks suspicious or makes no sense, you will also have an easier time finding the mistake in your own work if your work is organized. That can save you valuable time on an quiz or exam and may make the difference between finding and correcting your own mistake or leaving it to me to find it, which will cost you points. Even if you are doing your homework and are not under time pressure, it will save you frustration to be able to look back and find mistakes in your work easily because your writing is clear and organized. And if you did not make a mistake, that is great, but you will likely still come back to review your own work later, for example when you are preparing for a quiz/exam (this would be very wise to do), and you will also have an easier time following it if your writing is organized. Besides, in any kind of formal academic writing–and yes, your math homework and quizzes and exams are supposed to be examples of academic writing–the goal is to communicate your intellectual ideas to the reader effectively. So here we go.

1. Don't drop the limit sign until you have actually evaluated the limit or determined that the limit does not exist. For example, this is incorrect:

$$\text{Wrong:} \qquad \lim_{x\to 1} \frac{x^3-1}{x-1} = \frac{(x-1)(x^2+x+1)}{x-1} = \text{ etc}\ldots$$

   Correctly written, it would be

$$\text{Right:} \qquad \lim_{x\to 1} \frac{x^3-1}{x-1} = \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{x-1} = \lim_{x\to 1}(x^2+x+1) = 3.$$

   The first version is wrong because it's confusing the limit of a function with the function itself – they're not the same! Practically speaking, it leads to errors, because students who drop the limit sign early in a problem often forget, at the end, that they're supposed to do an evaluation step (like letting $x$ go to 1), because they've stopped writing that part. And from a reader's perspective, it makes the chain of equations harder to follow ("Weren't we taking a limit? What happened to that?").

   The flow of the correct version, when read aloud, is "This limit is equal to this limit, which is equal to this simpler limit, which we can finally evaluate by direct substitution: it's 3." It's a long sentence, but it is correct and meaningful, and not hard to follow.

2. Don't write "unattached" limit signs. For example,

$$\text{Wrong:} \qquad \lim_{x\to 1} = 4$$

   is an incorrect use of the notation, and is unacceptable. You might write

$$\text{Right:} \qquad \lim_{x\to 1} f(x) = 4 \qquad \text{or} \qquad \lim_{x\to 1} \frac{x+3}{x} = 4,$$

   but a limit is always a limit *of* something. In the wrong example, I would ask, "The limit of *what*, as $x$ approaches 1? What are you taking the limit of?" It's not clear.

   This causes difficulties, because a multistep problem may have more than one limit calculation in it. If you need to look back to find the value of a limit from an earlier step, a statement like the wrong example won't give you any clue what limit you were evaluating.

Every lim sign you use should have something beneath it (like $x \to 1$) to indicate what variable is approaching what point, and a function immediately following it to indicate what you're taking the limit *of*.

3. Don't write ungrammatical things, even when you're using mathematical symbols. For example,

$$\text{Wrong:} \qquad \lim_{x\to 2} \frac{1}{x-2} = \text{ does not exist}$$

would be read aloud as, "The limit of $\frac{1}{x-2}$, as $x$ approaches 2, equals does not exist." That's both wrong (the limit doesn't *equal* anything), and not even valid English. If you mean

$$\text{Right:} \qquad \lim_{x\to 2} \frac{1}{x-2} \text{ does not exist}$$

then say that! Or if you'd prefer to say

$$\text{Right:} \qquad \lim_{x\to 2} \frac{1}{x-2} \text{ is undefined}$$

that's a fine way to say it too. Both of those can read aloud as proper, grammatical sentences. The first version cannot.

Two more comments, broadly speaking (not just about limits):

4. If you can't read what you've written aloud, in grammatical English, then (1) it's probably nonsense, and (2) you've written something that you yourself don't understand, which can't possibly be acceptable to either of us.
5. Don't put the burden on the reader. When you're writing math for me, don't assume, "The professor knows how to do this problem, so he'll figure out what I mean even if I can't express it." Treat me as if I need things explained very clearly.

   Yes, I already know the answers. But I don't want to fish around in your work to see if you might have done something relevant to finding the answer. It's your responsibility to present your work clearly.

   **Your goal is to write a solution which is sufficiently clear and organized that someone who does not yet know how to solve the problem could learn to solve it by reading what you've written.**