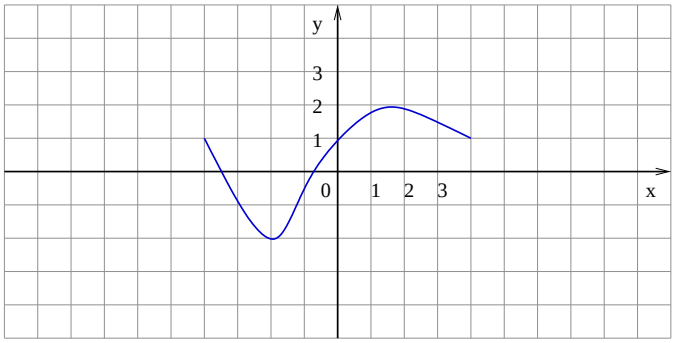
MCS 118 Exam 2

Nov 13, 2020

All of your answers must be carefully justified. Neat work, clear and to-the-point explanations will receive more credit than messy, chaotic answers. You may refer to any result proved in class unless otherwise specified. You may use results you proved on your homework, except for ones the problem specifically asks you to prove.

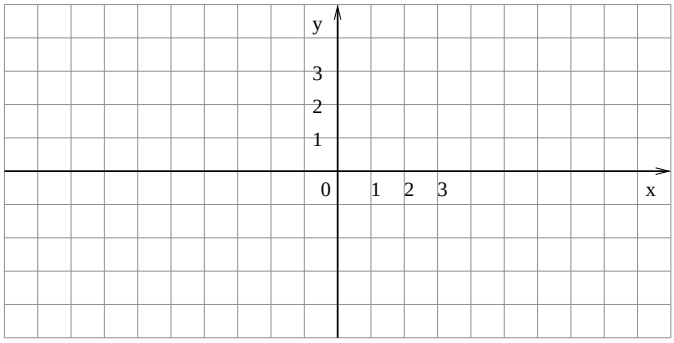
You are not allowed to use your textbook or your class notes, but you may use a simple calculator.

1. (5 pts each) Let $f(x) = \sqrt{3-x}$ and $g(x) = \sqrt{x^2-1}$.
   (a) Find the function fg and the largest possible subset of the real numbers that could be the domain of fg.
   (b) Find the function f/g and the largest possible subset of the real numbers that could be the domain of f/g.
2. (5 pts each) Let f and g be functions of real numbers such that f can be composed with g. Suppose g is an odd function and let $h = f \circ g$.
   (a) Is h always an odd function? If you think it is, prove that it is; if you think it is not, find a counterexample.
   (b) What if f is odd? Is h always an odd function then? If you think it is, prove that it is; if you think it is not, find a counterexample.
3. (5 pts each) The graph of a function $f(x)$ is given below.

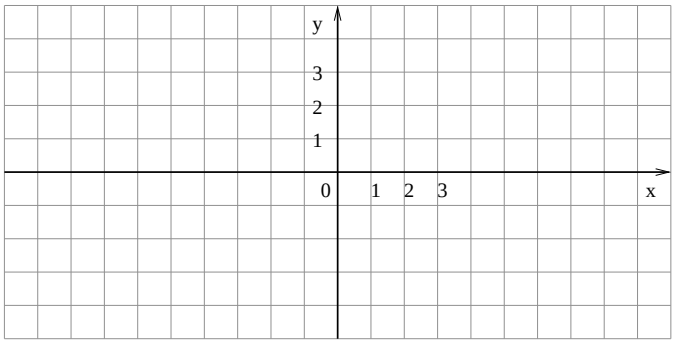


Sketch the graphs of the following functions. Make sure you explain your work.

(a) $g(x) = -2f(x-6)$



(b) $h(x) = f(2x-6)$



4. (a) (4 pts) Let f be a function of real numbers and $a \in \mathbb{R}$ such that in some neighborhood of a, $f(x)$ has a value for x except possibly at $x = a$. State the informal definition of

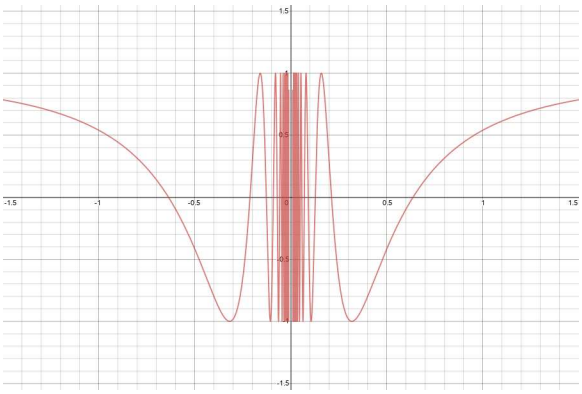
$$\lim_{x \to a} f(x) = L$$

where L is a real number.

(b) (6 pts) The graph of the function

$$f(x) = \cos\left(\frac{1}{x}\right)$$

is shown below.



Explain why

$$\lim_{x \to 0} \cos\left(\frac{1}{x}\right)$$

does not exist.

5. (5 pts each) **Extra credit problem.**

(a) Let f be an even function and let a be a real number such that

$$\lim_{x \to a^-} f(x) = L.$$

Does

$$\lim_{x \to -a^-} f(x) = L$$

have to be true?

(b) Now, let g be an odd function and let a be a real number such that

$$\lim_{x \to a^+} g(x) = L.$$

Does

$$\lim_{x \to -a^+} g(x) = -L$$

have to be true?