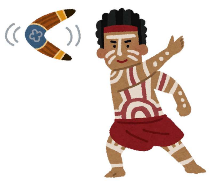
MCS 118 EXAM 2

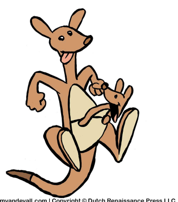
1. (10 pts) Let $f(x) = \frac{x}{1+x}$. Find the correct expression for the function $f \circ f(x)$ and the largest possible subset of the real numbers that could be the domain of $f \circ f$. Do not forget to justify your answers.

2. (10 pts) Let f and g be linear functions with equations $f(x) = m_1 x + b_1$ and $g(x) = m_2 x + b_2$. Is $f \circ g$ also a linear function? If so, what is the slope of its graph? Do not forget to justify your answers.



3. An Australian aboriginee throws a boomerang parallel to the ground. The distance of the boomerang from its owner at time t is given by the function $d(t) = -4t^2 + 30t$, where the distance is measured in meters and time is seconds. The boomerang barely misses a peacefully grazing kangaroo at $t = 2$.

   (a) (6 pts) Find the average velocity of the boomerang over the following time intervals:
      (i) $[2, 2.5]$
      (ii) $[1.9, 2]$
      (iii) $[2, 2.05]$
      (iv) $[1.99, 2]$



   (b) (4 pts) Use your results in part (a) to estimate the instantaneous velocity of the boomerang as it passes by the startled kangaroo at $t = 2$. Explain how you came up with your estimate.

4. (a) (4 pts) Let f be a function of real numbers and $a \in \mathbb{R}$ such that in some neighborhood of a, $f(x)$ has a value for x except possibly at $x = a$. State the informal definition of
$$\lim_{x \to a} f(x) = L$$
where L is a real number.

   (b) (6 pts) Explain why the function $f(x) = \frac{1}{x}$ does not have a limit as $x \to 0$.

5. (10 pts) **Extra credit problem.** Remember that a rational number is a number of the form m/n where m and n are integers and $n \neq 0$, and an irrational number is a real number that cannot be expressed as a ratio of two integers. For example, $2/3$ and $-13/5$ are rational numbers, but π and $\sqrt{2}$ are known to be irrational. One property (not essential for answering the question below) of rational/irrational numbers you may have encountered is that rational numbers have finite or infinite but repeating decimal expansions, whereas irrational numbers have infinite decimal expansions without a repeating pattern.

   Let $f : \mathbb{R} \to \mathbb{R}$ be the function
$$f(x) = \begin{cases} 1 & \text{if } x \text{ is a rational number} \\ -1 & \text{if } x \text{ is an irrational number.} \end{cases}$$
Does either
$$\lim_{x \to 0^+} f(x)$$
or
$$\lim_{x \to 0^-} f(x)$$
exist? If so, find its value; if it does not, explain why it does not exist.