MCS 118 Review Sheet

Here is a list of topics we have covered so far. Your review should certainly include your homework problems, as some of these will show up on the exam. You should also review the problems in your Webwork homework as the problem solving strategies you practiced while working on those may prove useful in solving problems on the exam.

The word "understand" is often used below. The definition of understanding is that you understand something when you know why it is true and can give a coherent and correct argument (proof) to convince someone else that it is true.

- Functions
  - Definition of a function, what makes a rule a function, examples and non-examples. Four standard ways to represent a function: verbal description, formula, table of values, graph.
  - Domain, codomain, and range.
  - Functions whose domain and codomain are subsets of $\mathbb{R}$.
    - The vertical line test.
    - Piecewise defined functions, examples, the absolute value function.
    - Even and odd functions, definitions, examples, symmetries of their graphs.
    - Increasing and decreasing functions, definitions, examples.
- A catalog of essential functions.
  - Linear functions, slope, y-intercept, how to find the equation of a line.
  - Power functions $f(x) = x^n$, how to make sense of positive and negative integer exponents, rational number exponents and roots.
  - Families of curves as the exponent changes in $f(x) = x^n$, shape of the graph if $n > 1$, if $0 < n < 1$, if $n < 0$.
  - Polynomials, coefficients, degree, leading term, graphs of polynomials, roots (zeros) of polynomials, factoring and how roots are related to factors.
  - Rational functions, domain of a rational function, graphs of rational functions, vertical asymptotes and other discontinuities.
  - Trigonometric functions, definitions in a right triangle, definitions extended via the unit circle, graphs of trig functions and their symmetries, periodic behavior, a few fundamental trig identities, such as $\cos(x) = \sin(\pi/2 - x)$, $\tan(x) = \sin(x)/\cos(x)$, and $\sin^2(x) + \cos^2(x) = 1$.
  - Exponential functions $f(x) = a^x$, shape of the graph for different values of the base, $a > 0$, irrational exponents.
  - Logarithmic functions, definition of $\log_a(x)$, connection to exponential functions, shape of the graph depending on a.
  - New functions from old functions
    - Transformations of functions: $g(x) = f(x) + c$, $g(x) = cf(x)$, $g(x) = f(x + c)$, and $g(x) = f(cx)$. Understand what these do to the graph of f depending on the value of c. Also, understand how you can combine these transformation (i.e. in what order to do the shift, stretch/compression, reflection, if the order matters).
    - Composition of functions, the meaning of $f \circ g$, when can you compose two functions and when can you not, how composition affects the domain.
- Limits of functions
  - The informal meaning of $\lim_{x\to a} f(x) = L$. Understand what it means that $f(x)$ gets arbitrarily close to L as x gets sufficiently close to a. Understand what it means when

the limit does not exist. Can you think of some examples of functions whose limits do not exist somewhere?
  - Guessing limits by substituting values of x near a into $f(x)$ and the dangers of this approach (remember $\lim_{x\to 0}\sin(\pi/x)$). Using the graph of f to guess the limit and the dangers of this approach.
  - One-sided limits, informal definitions of left and right limits. Understand what the informal definitions mean, how they are similar to the two-sided limit, and how they are different.
  - The formal definition of the limit. Understand what the $\delta - \epsilon$ definition means and how proving $\lim_{x\to a} f(x) = L$ using a $\delta - \epsilon$ argument is different from guessing the limit. Be able to construct $\delta - \epsilon$ arguments to prove $\lim_{x\to a} f(x) = L$ in some simple cases, such as for functions like $f(x) = 3x - 5$, $f(x) = x^2 - 3x + 1$, $f(x) = \sqrt{x}$, $f(x) = 4/x$, etc.
  - The formal $(\delta - \epsilon)$ definition of left and right limits. Understand what the definition means and how to use it to prove $\lim_{x\to a^-} f(x) = L$ or $\lim_{x\to a^+} f(x) = L$ in simple cases.
  - Understand how a function f that does not have a limit at a point a would fail to satisfy the $\delta - \epsilon$ definition. Can you think of an example?
  - The Limit Laws. Understand why they hold, how to use them, and when they cannot be used.
  - The Direct Substitution Property for limits of polynomials and rational functions. Understand why it holds (proof) when it can be used and when it cannot.
  - Replacing $f(x)$ with another function $g(x)$ in $\lim_{x\to a} f(x)$ as long as $f(x) = g(x)$ for all x except possibly at $x = a$. Understand why this is allowed and how it can be used to evaluate limits.
  - The connection between the two-sided limit and one-sided limits: $\lim_{x\to a} f(x)$ exists and equals a number L if and only if $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ both exist and equal L. Do you know how to prove this?
  - Theorem: If $f(x) \leq g(x)$ for every x in some neighborhood of a and $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist then $\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x)$. Understand why this is true.
  - The Squeeze Theorem. Understand how it can be used to evaluate limits.
  - Limits involving trigonometric functions, the limits of $\sin(x)$, $\cos(x)$, $\frac{\sin(x)}{x}$, $\frac{\cos(x)-1}{x}$ as $x \to 0$.
  - Using tools strategically to evaluate limits. Understand how the Limit Laws, the Direct Substitution Property, and other results about limits can be combined with appropriate algebraic manipulations (e.g. factoring, reducing and expanding fractions, rationalizing the numerator or denominator, etc) to evaluate limits.
  - The limit of a composite function: if $\lim_{x\to a} g(x) = b$ and $\lim_{x\to b} f(x) = L$, then $\lim_{x\to a} f(g(x)) = L$. Proof?

- Continuity
  - Definition of f being continuous at a in terms of the limit. Understand how this is the same as the Direct Substitution Property.
  - Types of discontinuities: removable, jump, and infinite discontinuities.
  - Left and right continuity, definitions, examples.
  - Continuity over an interval.
  - Understand why polynomials and rational functions are continuous at any number x in their domains.
  - Adding and subtracting continuous functions, multiplying by a scalar, multiplying, and dividing continuous functions.

- Polynomials, rational functions, root functions, and trigonometric functions are continuous at any number x in their domains.
- If g is continuous at a, then $\lim_{x\to a} f(g(x)) = f\big(\lim_{x\to a} g(x)\big)$. The composition of two continuous functions is continuous.
- The Intermediate Value Theorem, examples and applications.