# Notes for Section 3.3

This section summarizes a number of connections between the derivative of a function $f$ and the shape of the graph of $f$. We have already talked about quite a few of these, while others expand on ideas that will likely look familiar to you. For example, we already know that if a function is increasing over an interval, then its derivative is positive there, and if it is decreasing, then its derivative is negative. The section starts off by proving that the converse is also true: if the derivative of a function $f$ is positive (negative) over an interval then $f$ must be increasing (decreasing) over that the interval. The proof is an easy consequence of the MVT. (Remember that I said that the significance of the MVT is that it can be used in proving other results?)

The First Derivative Test–which also makes good intuitive sense if you give some thought–finally gives us a tool to tell if a critical point is a local maximum, minimum, or neither.

We have already talked about the connection between the second derivative and concavity and observed that if the graph of a function $f$ is concave up (down) over some interval, then $f''$ is positive (negative) there. The converse is also true: the sign of $f''$ over an interval tells you if the graph of $f$ is concave up or down over that interval. This gives us another tool called the Second Derivative Test to tell if a critical point $c$ such that $f'(c) = 0$ is a local maximum or minimum. What is says also makes good sense: if the graph of $f$ is concave up, then we have a local minimum at $c$, and if it is concave down, then we have a local maximum. The Second Derivative Test is not quite as powerful as the First Derivative Test because

- it only works for critical points of the kind $f'(c) = 0$ and not when $f'(c)$ does not exist,
- it only works reliably if $f''$ is continuous near the critical point $c$ (for our purposes in MCS 119, this is likely to be true),
- it can easily turn out to be inconclusive if $f''(c)$ turns out to be 0.

Despite all these weaknesses, the Second Derivative Test is often quicker or easier to do than the First Derivative Test.

Do not miss the definition of the inflection point. Next time the conversation at a party you are attending turns to inflection points, you don't want to be the only person in the room that does not have a good understanding of what an inflection point is.