Notes for Section 3.5

This section illustrates how we can apply what we know about the absolute and local extrema of functions in practical contexts by working through a number of examples. Such problems are called optimization problems and are some of the most typical applications of calculus. As usual, the biggest challenge in these problems tends to be translating words into formulas. The same general advice you were given in Section 2.7 still works. Other than that, it is a matter of practice. You will want to read through the examples and understand them. I will work through an example or two with you in class and I am glad to take requests on which example(s). You can pick one of the examples in the section or one of the exercises if you prefer. If you don't choose, I will.

Here are some notes for the examples in this section.

- Example 1 is a standard and straightforward example of using the Closed Interval Method we learned in Section 3.1. We are looking for the absolute maximum of a continuous function on a closed interval. So we find the critical points within the interval, and evaluate the function at those and at the endpoints. The largest value we find is the absolute maximum. Pay attention to what the problem asks you to find and make sure that is the question you answer. In this case, it does not want the largest area, but the dimensions of the field with the largest area.
- Example 2 shows you what you can do if you need to find the absolute extrema of a function over an open interval. The Closed Interval Method does not work here. We still approach the problem by finding the critical points within the interval. It turns out there is only one. We can show that it is an absolute minimum by arguing that the function is always decreasing before this critical point because its derivative is negative, and it is always increasing after the critical point because its derivative is positive. The book refers to this argument as The First Derivative Test for Absolute Extreme Values. Therefore its value at the critical point is smaller than anywhere else. Another valid argument would be that it is easy enough to see that the function is continuous on the open interval and that its values go towards ∞ as the input variable $r \to \infty$ and also as $r \to 0^+$. Therefore it must have an absolute minimum somewhere within the interval $(0, \infty)$, and that absolute minimum must also be a local minimum and hence a critical point. So it must be the one critical point we found.
- Example 3 is similar to Example 2 in that we are looking for the absolute minimum of a function over an open interval. The point of the example is to illustrate that the calculations can be simplified with a little thinking. While the problem could be solved by differentiating the distance function, the square of the distance, which must have its absolute minimum at the same place where the distance, is a simpler function, is easier to differentiate, and its critical points are easier to find.
- Example 4 is similar to Example 1 in that it is asking for the absolute minimum of a continuous function over a closed interval, so the Closed Interval Method can be used. Only the function is more complicated than in Example 1.
- Example 5 is another use of the Closed Interval Method, but it also illustrates that sometimes an absolute extremum can be found without using calculus at all by setting up the problem in a different way.
- Example 6 illustrates an application in business. It is a straightforward maximization problem and the solution uses the same ideas as Examples 1, 3, and 4. This problem could also be solved using the Closed Interval Method if the profit function were set up as a function of $y =$ the amount of the rebate. In this case, it would be clear that the only realistic values of y are in the closed interval $[0, 350]$. For fun, you could try solving it that way to see how that works out.