MCS 119 Exam 2

All of your answers must be carefully justified. Neat work, clear and to-the-point explanations will receive more credit than messy, chaotic answers. You may refer to any result proved in class unless otherwise specified. You may use results you proved on your homework, except for ones the problem specifically asks you to prove.

You are not allowed to use your textbook or your class notes, but you may use a simple calculator.

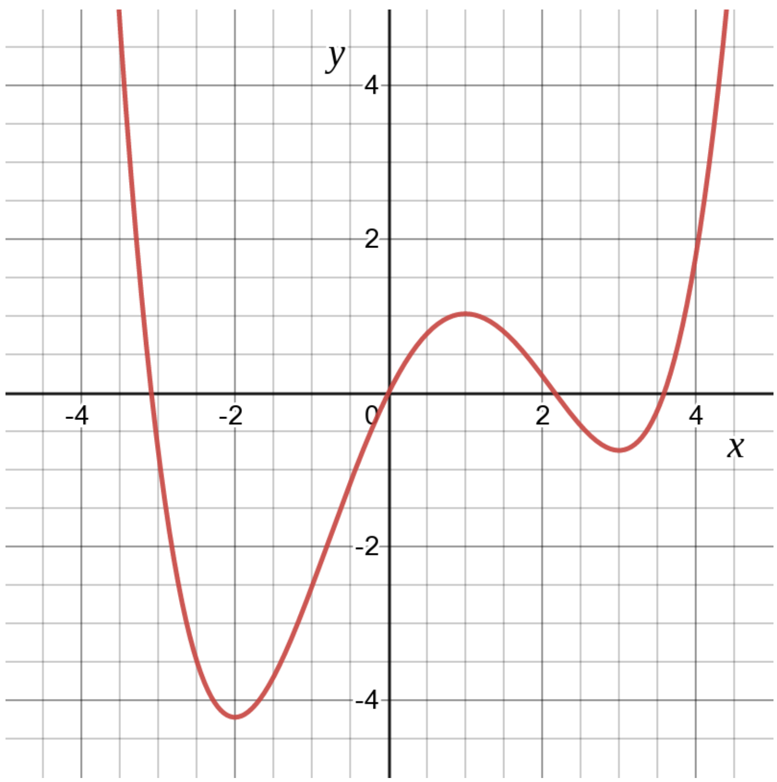
1. (10 pts) Find the n-th derivative of

$$f(x) = \frac{1}{x}$$

by calculating the first few derivatives and observing the pattern that occurs.

2. If f is a differentiable function, find an expression for the derivative of each of the following functions.
   (a) (4 pts) $y = \frac{x^2}{f(x)}$
   (b) (6 pts) $y = \frac{1+xf(x)}{\sqrt{x}}$

3. The graph of the function f is given below.



   (a) (7 pts) Sketch the graphs of f' and f''. You may do this on the same axes above.
   (b) (3 pts) For what values of x is $f'(x) = 0$? What can you say about f at those points?

4. (a) (3 pts) Let f be a function of real numbers and let a be a real number. What does it mean for f to be differentiable at a?

(b) (7 pts) Give a rigorous argument why the absolute value function $f(x) = |x|$ is not differentiable at 0.

5. (10 pts)**Extra credit problem.** Let f be a function of real numbers. The *symmetric derivative* of f at $x \in \mathbb{R}$ is defined as

$$\lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}.$$

If the symmetric derivative of f exists at x, then f is said to be *symmetrically differentiable* at x.

Prove that the absolute value function $f(x) = |x|$ is symmetrically differentiable at 0 by showing that its symmetric derivative exists there.