MCS 119 Exam 1

All of your answers must be carefully justified. Neat work, clear and to-the-point explanations will receive more credit than messy, chaotic answers. You may refer to any result proved in class unless otherwise specified. You may use results you proved on your homework, except for ones the problem specifically asks you to prove.

You are not allowed to use your textbook or your class notes, but you may use a simple calculator.

1. (10 pts) Use the definition of continuity and the properties of limits to show that the function
$$f(x) = 3x^4 - 5x + \sqrt[3]{x^2 + 4}$$
is continuous at 2.

2. (10 pts) Prove using the formal definition of the limit that
$$\lim_{x \to \infty} x^3 = \infty.$$
Hint: start by stating what the definition says when applied to this particular limit.

3. Beavis and Butthead play frog baseball. As Beavis pitches the (poor, unlucky) frog, Butthead observes that the position of the frog measured as its distance in feet from Beavis is given by the function
$$f(t) = 90t - 13t^2$$
where t is time in seconds and $t = 0$ is the moment Beavis releases the frog.

   (a) (6 pts) Use the definition of the derivative to find the instantaneous velocity of the frog at time $t = a$. What are the physical units of the derivative?

   (b) (4 pts) Use your result from part (a) to find the equation of the tangent line to f at $t = 3$.

4. (10 pts) Show that the function $f(x) = |x|$ is not differentiable at $x = 0$.

5. (5 pts each)**Extra credit problem.** Let f be a function of real numbers. The *symmetric derivative* of f at x is defined as
$$f_s(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}.$$
The function f is *symmetrically differentiable* at x if the limit above exists.

   (a) Prove that $f(x) = |x|$ is symmetrically differentiable at $x = 0$.

   (b) Find a function $f : \mathbb{R} \to \mathbb{R}$ and a real number x such that f is not *symmetrically differentiable* at x. Make sure you justify your answer by showing that f_s does not exist.