MCS 119 Exam 2

All of your answers must be carefully justified. Neat work, clear and to-the-point explanations will receive more credit than messy, chaotic answers. You may refer to any result proved in class unless otherwise specified. You may use results you proved on your homework, except for ones the problem specifically asks you to prove.

You are not allowed to use your textbook or your class notes, but you may use a simple calculator.

1. (10 pts) If $g(x) = x^{2/3}$, show that $g'(0)$ does not exist. Hint: note that you cannot do this by using the power rule because $\frac{d}{dx}x^{2/3} = \frac{2}{3}x^{-1/3}$ is not a valid formula at $x = 0$, but you can use the definition of the derivative and show that the limit does not exist.

2. (10 pts) Find equations of both lines through the point $(2, -3)$ that are tangent to the parabola $y = x^2 + x$.

3. (10 pts) Use the definition of the derivative to prove

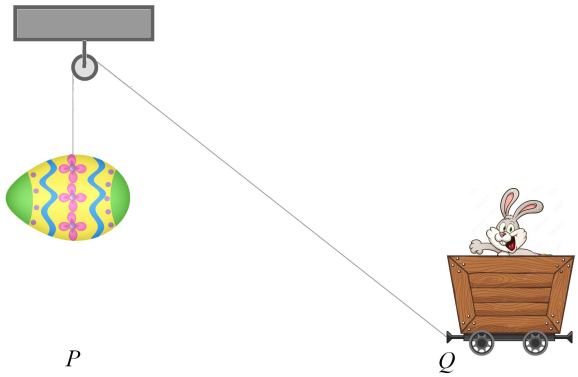
$$\frac{d}{dx}\sin(x) = \cos(x).$$

   Hint: the trigonometric identity

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

   may come in handy.

4. (10 pts) A popular ride in Easter Bunny Amusement Park consists of a cart tied to a 140-foot rope, which passes over a pulley at a height of 60 ft above the ground. The other end of the rope is tied to a humongous Easter egg. The weight of the egg pulls on the rope and moves the cart. When the cart is at a distance of 80 ft from the point directly below the pulley (point P in the diagram), the egg is falling toward the ground at a speed of 30 ft/s. How fast is the cart moving at the same time? The diagram below is definitely not drawn to scale.



5. **Extra credit problem.** Let n be a positive integer. If n is odd, then let x be any real number; if n is even, then let x be any nonnegative real number. By the power rule,

$$\frac{d}{dx}\sqrt[n]{x} = \frac{d}{dx}x^{\frac{1}{n}} = \frac{1}{n}x^{1/n-1} = \frac{1}{n}\frac{1}{x^{\frac{n-1}{n}}}.$$

   But we never proved in class that the power rule works for powers with fractional exponents. That is what you will do in this exercise. First, notice that if $y = \sqrt[n]{x}$ then $x = y^n$.

   (a) (5 pts) Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

(b) (5 pts) Substitute $y = \sqrt[n]{x}$ into $\frac{dy}{dx}$ to express $\frac{dy}{dx}$ only in terms of x. Did you get the expected result?