

MCS 150 HOMEWORK 10

1. Let A and B be sets. We defined the Cartesian product $A \times B$ as the set of ordered pairs $\{(a, b) \mid a \in A, b \in B\}$. Notice that if A or B is the empty set, the Cartesian product is also empty.
 - (a) Show that if A and B are finite then $|A \times B| = |A| |B|$.
 - (b) Show that if A and B are both countable (finite or infinite), then $A \times B$ is also countable.
2. (a) Let $1 \leq k \leq n$ be integers. Prove that

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}.$$

- (b) Prove the Binomial Theorem

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$$

by induction on n .

3. Define the Fibonacci Sequence by

$$F_0 = 0$$

$$F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2} \qquad \text{for } n \geq 2.$$

Show that if n is a nonnegative integer, then

$$\sum_{i=0}^n F_i^2 = F_n F_{n+1}.$$