MCS 150 Homework 2

1. Prove or disprove (e.g. by finding a counterexample).
   (a) If n is an integer such that $2|n$ and $3|n$ then $6|n$.
   (b) If n is an integer such that $4|n$ and $6|n$ then $24|n$.
2. Let x, y be odd integers.
   (a) Prove that $4|(x^2 - y^2)$.
   (b) Prove that $8|(x^2 - y^2)$.
   (c) Prove that $4 \nmid (x^2 + y^2)$.
3. Prove the following propositions.
   (a) Let $a, b, c \in \mathbb{Z}$. If $a|b$ and $b|c$ then $a|c$.
   (b) Let $a, b, c \in \mathbb{Z}$. If $a|b$ and $a|c$ then $a|kb + nc$ for any $k, n \in \mathbb{Z}$.
   (c) Let $a, b, c, d \in \mathbb{Z}$. If $a|b$ and $c|d$ then $(ac)|(bd)$.
4. The Division Algorithm says that if $n \in Z$ and $d \in \mathbb{Z}^+$, then there exist unique integers q, r such that $n = qd + r$ where $0 \leq r < d$. Give a proof.