# MCS 150 Homework 3

1. Let $m, n \in \mathbb{Z}$ not both 0. We defined the greatest common divisor of $m$ and $n$ as a positive integer $d$ such that
   (a) $d|m$ and $d|n$,
   (b) if $c$ is an integer such that $c|m$ and $c|n$ then $c|d$.
   Prove that the greatest common divisor is unique. That is if $d$ and $d'$ both satisfy the two conditions above, then $d = d'$.
2. In this exercise, you will prove that our definition of the greatest common divisor is equivalent to the one you learned in middle school. Let $m, n \in \mathbb{Z}$ not both 0.
   Suppose $d_1 > 0$ is the greatest common divisor of $m$ and $n$ according to our definition:
   (a) $d_1|m$ and $d_1|n$,
   (b) if $c$ is an integer such that $c|m$ and $c|n$ then $c|d_1$.
   Let $d_2$ be the common divisor of $m$ and $n$ that is the largest number among the common divisors. That is we know
   (a) $d_2|m$ and $d_2|n$,
   (b) if $c$ is an integer such that $c|m$ and $c|n$ then $c \leq d_2$.
   Show that $d_1 = d_2$.
3. Let $a, b \in \mathbb{Z}$ not both 0. Remember that we say $a$ and $b$ are relatively prime if $\gcd(a, b) = 1$. Prove that $a$ and $b$ are relatively prime if and only if there exist $m, n \in \mathbb{Z}$ such that $ma + nb = 1$.