MCS 150 Homework 5

1. Let $n \in \mathbb{Z}^+$. The *additive order* of an integer x modulo n is defined as the least $k \in \mathbb{Z}^+$ such that
$$\underbrace{x + x + \ldots + x}_{k \text{ times}} \equiv 0 \pmod{n}.$$
That is k is the least positive integer such that $kx \equiv 0 \pmod{n}$. Formulate a conjecture about what k is in terms of x and n. Prove your conjecture. (Hint: experiment a little with some examples.)

2. Let $n \in \mathbb{Z}^+$ and $x, y \in \mathbb{Z}$. Show that if x and y are both invertible modulo n, then xy is also invertible modulo n. This result says that the set of units modulo n is closed under multiplication. That is if you multiply two elements of this set together you get another element of this set.

3. Let $n \in \mathbb{Z}^+$. We remarked in class that if $n = p$ a prime, then every integer that is not congruent to 0 modulo n is a unit. That is among the canonical representatives $0, 1, \ldots, p-1$, all but one is a unit. Suppose $n = p^k$ where p is a prime number and $k \in \mathbb{Z}^+$. What can you say now about the number of units among the canonical representatives $0, 1, \ldots, n-1$?