Some counting problems

By word below, we mean a string of letters. It does not need be meaningful or even pronounceable.

1. How many five-letter words can you form using the standard English alphabet?

2. How many five-letter words can you form using the standard English alphabet if every word must have exactly one vowel? What if every word must have exactly two vowels?

3. How many five-letter words can you form using the standard English alphabet if every word must have at least one vowel?

4. Given 26 Scrabble tiles with the letters or the English alphabet, how many five-letter words can you form? What if each word must have exactly two vowels?

5. A lottery game uses the integers $1, \ldots, 90$. You choose five of these numbers on your lottery ticket and win the first prize if your numbers match the five numbers selected in a random drawing. How many ways do you have to choose the five numbers on your lottery ticket? What is the probability that you win the first prize if you play with one lottery ticket?

6. Same lottery game as in the previous problem. You win second prize if four of the numbers on your lottery ticket match any four of the five numbers randomly drawn. What are the odds that your lottery ticket wins second prize?

7. How many different ways are there to deal five cards from a standard 52-card deck?

8. How many five-card hands dealt from a standard 52-card deck will contain at least one pair of cards? Exactly one pair of cards? A full house (one pair and a three-of-a-kind)? A flush (five cards of the same suit)? A straight (five consecutive cards of any suit)?

9. Let S be a set with n elements. How many different k-element subsets does S have?

10. Given three red, four blue, and five green marble, how many different ways can you order them in a line?

11. Santa slides down the chimney of a house and finds four stockings hanging by the fireplace. Santa has 13 identical candy bars in his bag. How many ways can he divide the 13 candy bars among the four stockings if he wants to give everyone at least one candy bar?

12. Same setup as the previous question, but Santa is kind of mean and does not mind giving someone no candy at all. How many ways can he now divide the 13 candy bars among the four stockings?